Investigating how to design interactive learning environments to support students' learning of upper secondary and university math

Sanela MEHANOVIC[a*], Daniel SPIKOL[b]
[a]*Department of Mathematical Sciences, University of Gothenburg, Sweden*
[b]*Department of Computer Science, Malmö University, Sweden*
*sanela.mehanovic@chalmers.se

Abstract: Students' difficulties in learning of mathematics have for a long time, been investigated by researchers in different fields. Within educational research there are claims that technological tools appropriately integrated in students' mathematical work can support their understanding of a wide range of concepts in mathematics. This paper reports on the initial investigation for the design of Interactive Learning Environments (ILE) to support students' learning of mathematics. The project is guided by the notion of Design Based Research (DBR) and aims to explore how to design ILE that support students' understanding of integrals in particular. The initial study was conducted at a Swedish university with 10 students in 4 groups. The study confirmed difficulties in students' understanding of integrals as reported in educational literature and provides a set of design aims for the next iteration of the ILE to support the learning.

Keywords: mathematics, education, students, interactive learning environments

1. Introduction

Society needs a well-educated population, who not only actively contributes to the shaping of the society itself, but who also, as a broadly qualified work force is able to activate and transfer school content knowledge, insights, and skills to a variety of situations and contexts. Mathematics, from a societal perspective is recognized as one of the key components in this process, has lately met considerable difficulties. Schools and universities across the world meet with an increasing problem with young people having difficulties in dealing with mathematical content. The use of computers in mathematics education has often been an underlying goal of presenting mathematical concepts to students in a new and dynamic way compared to previous learning environments. Some mathematical concepts are difficult for students to understand when presented in the paper/pencil based teaching lend themselves to computer representations as in the case with the integral concept [1]. Integrals have visual aspects that can be displayed on a computer screen along the other representations such as algorithmic, symbolic, numerical, or natural language representations.

With the use of mathematical software for visualization, the notion of integrals is more easily adopted by students [2]. On the other hand, it also makes the didactical situation more complex [3]. A technological tool that becomes a mathematic work tool in the hands of the students is a process that has turned up unexpectedly complex [4]. The process causes differentiation in students' work with technological tools, meaning that different students have different experiences and work differently with the same tool and within the same environment. Furthermore, the work of Guin and Trouche [5] argues the more complex environment the larger the differentiation of students' work methods with these applications can result in more diverse learning trajectories.

1.1 Research Aim

This paper reports on the initial investigation in the design of an interactive learning environment (ILE) to support students' understanding of integrals. The project is guided by the notions of design-based research (DBR). In education DBR is used to develop and investigate (content oriented) theories though iterative cycles of intervention and refinement. DBR aims to combine the intentional design of interactive learning environments with the empirical exploration of our understanding of these environments and how they interact with the individuals [6].

The research aim is to explore how to design ILE that support understanding integrals in particular. In our opinion, a design of ILE should consider following two aspects: Firstly, it should attempt to minimize issues related to students' difficulties to optimally use technological tools in their mathematical work. Secondly, the design should, in parallel, deal with difficulties in students' understanding of a particular mathematical content.

2. Background

In the upper secondary education, the integral is generally defined in the following way (see figure 1): Let f(x) be a continuous function in a closed interval [a, b] divided in subintervals with equal length □x. Then, for n subintervals we have the area $S \sim \sum_{i=1}^{n} f(xi) \,\Delta\, xi$. If we let $n \to \infty$ then □$xi \to 0$ and it can be shown that $\sum_{i=1}^{n} f(xi) \,\Delta\, xi$ approaches a limit called the integral of f from a to b, which is denoted $\int_a^b f(x)dx$.

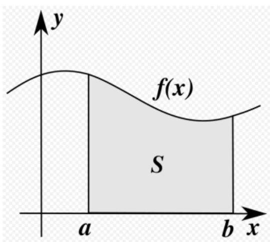

Figure 1: Integral defined as an area under the function f(x).

This definition based on a Riemann sum is difficult for students to understand. Students' difficulties with integrals are not a new behavior in the mathematics classroom and had, for the last several decades have been a subject for educational research [1,7,8]. In the early eighties, Orton observed that students had difficulties while solving tasks related to the understanding of integration as limit of sums [9]. In this study, students were able to apply, with some facility, the basic techniques of integration but further probing indicated fundamental misunderstanding about the underlying concepts. Students interpreted the integral as a procedure that transforms an input into some output. The same study revealed that students' technical ability could be quite strong, despite showing minimal conceptual understanding. Apart from showing strong procedural skills the students were found to demonstrate a strong reluctance to using geometric interpretations to complete an algebraic process, and when possible, were more inclined to move to an algebraic context [8].

Another study from Orton (1980) revealed that students had problems with the integral $\int_a^b f(x)dx$ if f(x) is negative or b is less than a [9].

More recent studies specialized in mathematics education show that this concept is still difficult for students' to grasp; they are not able to write meaningfully about the definition of a definite integral nor can they without difficulties interpret problems calculating areas and definite integrals in wider contexts [10]. The students also intend to identify the definite integral as an area [11].

2.1 Theoretical Framing

Students' understanding of integrals can be discussed from the perspective of the cognitive structure in their mind that is associated with the concept of integrals. Tall and Vinner [12] formulate a distinction between the mathematical concept as formally defined and the cognitive processes by which they are conceived by the students (p.1). The total cognitive structure that is associated with the concept, including all the mental pictures and associated processes, they name a concept image, and mean that a student's image of a mathematical concept may not be globally coherent and may have aspects which are quite different from its formal mathematical definition" (p.1). At different times, seemingly different conflicting images may be activated. The conflicting aspects, that are a part of a student's concept image and/or a concept definition, are called cognitive conflict factors (p.3). As a student does not necessarily see a conflict while using different methods in their mathematical work, the student simply utilizes the method he or she considers appropriate on each occasion [12]. The conflicting aspects that are a part of a student's concept image and/or a concept definition are called cognitive conflict factors (p.3). Only when conflicting aspects are evoked simultaneously need there be any actual sense of conflict or confusion (p.2).

3. Methods

Our initial study was conducted to investigate students' concept image of integrals, here in terms of the definite integral, in a way the concept is usually introduced to students (see figure 1). The study was conducted at a Swedish university and considered an introductory course in mathematics with 10 students. Four groups were self-created with 2 to 3 students for the intervention. The participating students, who were just about to finish their introductory course already had an image of the integral concept, were asked to take a test containing integral tasks based on a previous research conducted by Rolka & Rösken [13]. This test was developed in order to investigate students' understanding of the formal mathematical definition of the definite integral, and focused on aspects in integrals known to be difficult for students to grasp.

In the intervention, the students were supposed to within an hour, solve a test with eight integral tasks. They were asked to solve tasks in the test as a group, and to write their solution on a whiteboard while discussing a particular task. Their work was videotaped, and, once they agreed on a solution to a particular task, we took a photograph of their whiteboard notices. We are currently working on the analysis the video data and the solutions gathered from observing the groups of students.

4. Results

The initial study confirmed previous research within mathematics education. While solving the integral tasks, students have not always been aware of their conflicting images of the definite integral in relation to its definition. For instance, one of the images that has been shown to be highly present in the experiment group was the perception of the definite integral as an area, see figure 2.

In the task shown in Figure 2 (left image) students' were asked to calculate the value of the integral. The task considers the oriented area aspect of the integral and the results confirmed Rolka & Rösken's finding in which many students just equals the concepts of integral and area [13]. None of the four groups in our initial study came with a correct answer to this task. Three groups have chosen the option A as the final answer to the task. Only students in the remaining group considered that the correct answer might be another option than A. Indeed, they did suggest the correct option (B) although it has been suggested as a second alternative (even this group had A as their first choice).

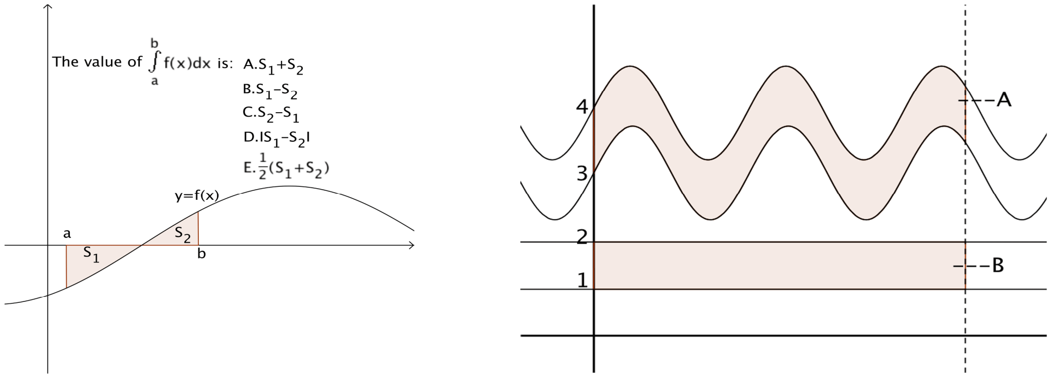

Figure 2: Examples of the students' perceptions of integrals.

Another task that all participating groups had difficulty with, was to deal with the problem illustrated in figure 2 (right side). The picture shows two areas A and B. What do you think is correct for the relation between the areas?

- The area of A is bigger than the one of B.
- The area of A is smaller than the one of B.
- Both areas are equal.
- Without any function given explicitly, it is not possible to answer this question

None of the groups had answers that they were certain about, rather, they were discussing different options having difficulties in choosing between the first and the third option. What was of a particular interest for the study was the contrast between these two options that seemed to cause a cognitive conflict for some of the students. The discussed aspect was following: The first option feels true instinctively, if one thinks that the area of A can be stretched outside of the interval (still keeping the same height). On the other hand, in the definition of the definite integral as a Riemann, the area between two curves is calculated as a sum of areas of infinitely thin rectangles. Only one student started to discuss the Riemann sum which led the whole group to move their reasoning to what answer option could be appropriate from the formal definition's perspective. In our opinion, this example demonstrates how conflicting images evoked simultaneously in students' work with a mathematical concept can lead them to a deeper reasoning of the meaning of the formal concept definition.

5. Discussion

The results of our initial exploration point to design goals for the ILE that include further investigations of the role of the technological tools for students' mathematical work with integrals. Misconceptions about the mathematical problem observed in the student groups point to the need for support. From our investigation of students' understanding of integrals and from the supporting research literature we are able to identify some implications for the next iteration of our ILE. In order to support the students to expose their conflicting images of integrals and gain a deeper understanding of this mathematical concept the following design goals have been identified:

- The ILE needs to guide the students though a learning process that exposes their concept image of integrals and then supports its' development, while taking into consideration a diversion in students' individual perceptions.
- This guiding process needs to provide support for individual learning exploration for the student though some types of externalization like adaptive scaffolding and teachable agents [14].
- Minimize issues related to students' difficulties to optimally use technological tools in their mathematical work by acting as a support component for learning instead of providing a wealth of features that can be seen at times to distract lower performing students [4].

For the next steps, we are currently designing a low-fidelity prototype that will explore teachable agent like qualities that we can test during a summer school mathematics class for university students. These students will form a new test group for the next round of participatory workshops with the teachers and researchers.

References

[1] Tall, D. (1991). Visualizing Differentials in Integration to Picture the Fundamental Theorem of Calculus. *Mathematics Teaching*, 137, 29-32.

[2] Herceg, D. & Herceg D. (2009). The Definite Integral and Computers. *The Teaching of Mathematics*, 12 (1), 33-44.

[3] Blomhøj, M. (2005). Villkor för lärande i en datorbaserad matematikundervisning - elevernas användning av avancerade matematikprogram. In B. Grevholm (Ed), *Matematikdidaktik - ett nordiskt perspektiv*, (pp. 185-218). Lund: Studentlitteratur.

[4] Artique, M.(2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7(3), 245-274.

[5] Guin, D. & Trouche, L. (2002). Mastering by the teacher of the instrumental genesis in *CAS environments: necessity of instrumental orchestration*. ZDM 2002 34 (5).

[6] Hoadley, C. M. (2004). Methodological alignment in design-based research. *Educational Psychologist*, 39(4), 203–212. Routledge.

[7] Orton, A. (1980). An investigation into the understanding of elementary calculus in adolescents and young adults. *Cognitive Development Research in Science and Mathematics*, 201-215. University of Leads.

[8] Machin, M.C. & Rivero, R.D. (2003). Using Derive to understand the concept of definite integral. *International Journal for Mathematics Teaching and Learning*, 1-16. ISSN 1 473-0111.

[9] Orton, A. (1983). Students' understanding of integration. *Educational Studies in Mathematics*, 14, 1-18. Dordrecht, Holland, and Boston, U.S.A.

[10] Rasslan, S. & Tall, D. (2002). Definitions and images for the definite integral concept. In Cockburn, A. D. Cockburn Nardi, E. (Eds), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, (Norwich, UK), 4, 89-96.

[11] Attorps, I., Björk, K. & Radic, M. (2011). The use of mathematics software in university mathematics teaching. Paperpresented at CERME 7, T*he Seventh Congress of the European Society for Research in Mathematics Education*, 9-13th February 2011. Rzeszow, Poland.

[12] Tall, D. & Vinner, S. (1981). Concept Image and Concept Definition in Mathematics with particular reference to Limits and Continuity. *Educational Studies in Mathematics*, 12, 151-169.

[13] Rösken, B. & Rolka, K. (2007). Integrating Intuition: the Role of Concept Image and Concept Definition for Students' Learning of Integral Calculus. *The Montana Mathematics Enthusiast*, ISSN 1551-3440, Monograph 3, 181-204. The Montana Council of Teachers of Mathematics.

[14] Blair, K., Schwartz, D., Biswas, G. and Leelawong, K.(2011). Pedagogical Agents for Learning by Teaching: Teachable Agents. To appear in *Educational Technology*.