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Epistemological aspects of multilingualism in mathematics education: an inferentialist approach

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ABSTRACT
Recently the prevailing language-as-resource metaphor has been problematised and theorised. Using the philosophical theory of inferentialism, we trace an epistemological dimension of multilingualism in mathematics education and add it to the current language-as-resource discussions. With data from two different settings—a mathematics classroom in Sweden and a workshop in an indigenous settlement in Colombia—we show that in encounters between language practices and plural mathematics, the semantic and the epistemological are two sides of the same coin. Inferentialism captures such encounters without dichotomising either languages or mathematics. We contend that epistemological issues move beyond the scope of language-as-resource approaches, but they are not paths to improving school achievement. Neither are they matters of distinguishing between formal and informal language use. Rather, an epistemological dimension is about shaping meta-understandings of language diversity that are liberated from mathematics as fixed and pre-established knowledge.

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Introduction
Researchers in mathematics education have developed various approaches to negotiating cultural and language diversity in classrooms. While diversity has existed since the beginning of schooling, research on language diversity in mathematics classrooms began in the 1970s. Multilingualism was approached as a deficiency that hinders student achievement (Phakeng, 2016). The response was to enhance student proficiency in the language of instruction. In the 1980s, framed by globalisation, migration and political rights struggles, a new strategy to “solve the problem”, as stated by Planas and Civil (2013), was to recognise people’s language rights and allow students’ first languages (L1s) to be an additional language of learning and teaching. Discourses on students’ L1s as a resource—the language-as-resource metaphor—in the learning and teaching of mathematics emerged and still underpin much of contemporary mathematics education research on multilingualism (Barwell, 2018; Chronaki & Planas, 2018). For instance, researchers have promoted
the pedagogical use of students’ first languages in flexible and strategic ways to strengthen communication, foster participation in mathematical activities, and, ultimately, create learning opportunities (Planas & Setati-Phakeng, 2014). The language-as-resource metaphor serves to move beyond the dichotomy of “language as problem vs. language as right” (Ruiz, 1984), which is ultimately a monolingual dichotomy (the use of only L2 or only L1). Although to think of students’ first language as a resource instead of a deficit still is a pressing issue in mathematics education (Norén & Andersson, 2016), to embrace language diversity as a pedagogical resource has been widely accepted (Chronaki & Planas, 2018).

A first wave of research in language-as-resource has become a leading trend during the past decade (see, e.g. Barwell, 2009; Bose & Clarkson, 2016; Moschkovich, 2008, 2015; Planas, 2014; Planas & Civil, 2013; Rubinstein-Ávila, Sox, Kaplan, & McGraw, 2015; Setati & Adler, 2000). Recently, researchers in a second wave have problematised (Parra & Trinick, 2018) and theoretically elaborated on (Barwell, 2018; Chronaki & Planas, 2018; Planas, 2018) the language-as-resource metaphor. In this paper, we join those efforts to explore the boundaries of the language-as-resource metaphor by grasping another aspect of the complexity in today’s multilingual mathematics classrooms. We argue that approaching multilingualism from a translanguaging perspective raises not only language but also epistemological concerns; “[t]ranslanguing is the enaction of language practices that use different histories [cultures and mathematics], but that now are experienced against each other in speakers interactions as one new whole” (García & Wei, 2014, p. 21, italics in original).

In this paper, we use illustrations from two different episodes—a workshop held during a revitalisation project in Colombian indigenous communities and a Grade 5 mathematics classroom in Sweden—to show how ideas based on the neo-pragmatic theory of inferentialism (Brandom, 1994, 2000) can illuminate epistemological dimensions in current multilingual mathematics educational contexts. We focus on the “construction and use of original and complex interrelated discursive practices” (García & Wei, 2014, p. 22) that emerge in interlocutors’ face-to-face translanguaging conversations. Our intention is to trace the existence of an epistemological dimension of multilingualism—an aspect that has not been the central focus in the recent development of the language-as-resource metaphor. To focus on the epistemological dimension, in this paper we look at the ways in which interlocutors (students and teachers) in social- and language-diverse educational contexts use and treat each other’s claims as they grapple with conveying meaning and shaping understandings of the world.

We have organised the paper as follows: first, we briefly address the rationale, constructs and operationalisation of the language-as-resource metaphor in what we consider its first wave. Then we review some aspects of three recent papers from the emerging second wave (Barwell, 2018; Chronaki & Planas, 2018; Planas, 2018) that problematise and theoretically elaborate on the language-as-resource metaphor in order to outline the need for an epistemological dimension. We then show how semantic and epistemological dimensions are two sides of the same coin when we abandon the idea of language mirroring the world “as it is” (i.e. a representation paradigm) (Brandom, 2008; Derr, 2013). We show that this insight is theoretically anchored in inferentialism. We then use inferentialism to elaborate on the emergence of an epistemological dimension and its social premises. We conclude the paper by proposing to include the epistemological issues that arise when rationalities of various languages meet as a central part of the
teaching and learning of mathematics. We propose a new focus for mathematics education research on multilingualism that is not achievable under the current framework of the language-as-resource metaphor. We do not adhere to the language-as-resource metaphor; rather, we offer an alternative stance that embraces mathematics classrooms as spaces where different forms of knowledge meet and, consequently, teachers and students learn to manage the plurality of languages and mathematics.

**Language diversity and the language-as-resource metaphor in mathematics education research**

The idea of language as a resource was introduced by Ruiz (1984), who suggested this metaphor as an alternative stance to predominant ideas about minority language use either as a right or as a problem, when developing a language-planning program in the United States. For minority languages to be useful, they had to be considered beneficial resources. Consequently, the idea of language-as-resource “presupposes that language can be valued as an asset” (Ricento, 2005, p. 361). Petrovic (2005) located language-as-resource approaches as part of the neoliberal agenda, raising the concern that “forces of neoliberalism are far more likely to be successful at manipulating diversity to maximise profit than cultural pluralists will be at manipulating neoliberalism to maximise diversity” (p. 410). Thus the original rationale of language-as-resource was part of a deficitarian approach, where the use of minority languages in the classroom was conceived of as a “temporary vehicle to promote access to full civic participation via the national language … and not to promote full-blown bilingualism/biliteracy” (Ricento, 2005, p. 361). In the mathematics classroom, this idea could be interpreted as a commodification of multilingual students’ L1s as a means to realise their transition from informal mathematics talk (L1) to formal school mathematics talk in the language of instruction (L2). From a national perspective, students’ L1s become strategic resources in the pursuit of economic growth as they are used in the “construction of techniques to relate individual achievement in math with national and international economic development” (Valero, 2017, p. 2), an initiative manifested in testing programs such as the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). The instrumentalisation of language described above downplays other essential aspects such as “psychological, cultural, affiliational, aesthetic and historical [ones], among others” (Ricento, 2005, p. 362). Consequently, to emphasise language as a pedagogical resource in mathematics classroom orchestration (Planas, 2012) fails to consider language diversity in its broader cultural, social, political and historical context.

**The first wave of language-as-resource**

The language-as-resource metaphor, which was explicitly brought into mathematics education research by Planas and Setati-Phakeng (Barwell, 2016), is generally operationalised by characterising multilingual students’ mathematics learning as a “journey from informal talk in their main language towards formal mathematical talk in [the language of instruction]” (Webb & Webb, 2016, p. 198). This idea underpins major strands in mathematics education research on multilingualism (see, e.g. Bose & Clarkson, 2016; Setati & Adler, 2000). Planas and Civil (2013) claimed that to use students’ L1s as a resource, it is necessary to have flexible views on language use. Researchers have suggested methods such as
code switching, peer talk, re-voicing and the use of synonyms, gestures and objects (Moschkovich, 2008, 2015; Planas, 2014; Planas & Civil, 2013) to support students’ participation in learning opportunities. Planas and Setati-Phakeng (2014) asserted that “[t]he extent to which language use facilitates communication and participation … is the extent to which learning is facilitated” (p. 885). The language-as-resource metaphor is an organising principle for classroom practices with the aim of achieving learning opportunities by integrating the foci on mathematics and language; it deals with tensions between language learning and the learning of mathematics (Barwell, 2012; Planas & Setati-Phakeng, 2014; Prediger & Krägeloh, 2016). Although the language-as-resource metaphor still enmeshes pressing issues (Norén & Andersson, 2016; Planas, 2018), it appears to be operationalised primarily as a tool for providing opportunities for emergent multilingual students to learn school mathematics and/or the language of instruction. We conclude that conceptualisations of the first language-as-resource wave in mathematics education research primarily “focus on the instrumental values of heritage languages while ignoring (or downplaying) the human beings, communities, and socio-political dimensions of language acquisition, use and loss” (Ricento, 2005). Language becomes a thing in itself, bounded and clearly defined as national languages referred to as L1 or L2. Moreover, first wave language-as-resource conceptualisations appear to be based on the idea that language conveys pre-established meanings detached from social, cultural, political and epistemological aspects inherent in language.

**An emerging second wave**

Recently, some scholars have begun to problematise the language-as-resource metaphor and offer alternative stances. Chronaki and Planas (2018) have challenged the representational paradigm of language. This paradigm starts from the idea that language mirrors the world “as it is” and conveys pre-established meanings. Inherent in the representational paradigm in mathematics education (see, for example Duval, 2006) is a tacit ontological assumption that there is but one global mathematics that originates from Western cultural traditions. This assumption implicitly excludes flexible plural views of mathematics and hence prevents students from producing mathematics-based discursive spaces that incorporate plural and diverse mathematics.

In contrast to the representational paradigm, Barwell (2018) and Planas (2018) recognised translanguaging; this specifies language use as socially shaping (stratifying) meaning and recognises that semantic meaning articulates the language use of other speakers.

This discursive dimension of multiple interrelated sources of meaning that multilingual students use when engaging with mathematics tasks can include mathematics as being socially and culturally multiple (Barwell, 2018). However, it is the distinction between formal and informal talk about the same mathematics that comes to the forefront. In a similar way, Planas (2018) took into account dialectic tensions between, on the one hand, the one global school mathematics and the language of instruction and, on the other, the multi-discoursed languages and mathematics (as a plural noun) of the learners (and their teacher). In small group work, she claimed, students move back and forth in the framework of tensions as they use language as a resource to realise some of the potential mathematical meanings embedded in the immediate situation. Planas (2018) denoted potential mathematical meanings.
as the substance of what is taught and ought to be learned in cultures of schooling and especially of school mathematics, recurrently shown and invoked as valued institutionalized ends in curriculum, task design and implementation, evaluation, assessment policies, and practices. (p. 220)

While the second wave of language-as-resource certainly offers a more nuanced approach to language diversity and mathematics than the first wave, we find that it still relies on an enthroned type of mathematics (the school version of the scientific discipline of mathematics) that cannot be questioned nor changed. Researchers continue to frame the school as the space in which students must achieve skills in that type of mathematics. Ideas of people having or lacking, for instance, language or mathematical proficiency are propagated. We therefore fear that the emerging second wave of the language-as-resource approach might become engulfed in the deficitarian paradigm also. In the next section, we consider the epistemological dimension as an alternative that envisions school activities as open spaces for meeting different and contrasting performances, allowing students to widen and interconnect their own mathematical knowledge and come to know those of the others.

**An inferentialist view of discourse, epistemology and multilingualism**

Breaking free from dichotomisations between different languages and mathematics and also ideas on students’ mastery of the two calls for different ways of thinking and reasoning about language and mathematics. The way of reasoning we propose here is based on expressivism. Expressivism uses discursive reasoning practices to make explicit the socially inferential rules in our language. Making implicit rules explicit enables us to reflect on them and change them. Using language and making meaning explicit sensitises our semantic consciousness (Arazim, 2017). Expressivism, where inferentialism resides, is not simply about expressing something that is already there. Expressing it also gives it a shape. Although there is freedom in the shaping, there is not anarchy; the freedom is governed by our semantic consciousness (Arazim, 2017). The idea of semantic consciousness embraces questions about what and how mathematics is expressed and shaped in multilingual contexts, and how students and teachers become sensitised to the diversity of language and mathematics. It moves beyond questions about how language is a resource for realising (Western) mathematics.

The above discussion brings us to the connection between the semantic and the epistemological. Before addressing that connection, we want to clarify that in this paper we understand epistemology to be a theory of knowledge that studies how humans and their cultures come to take something to be knowledge and the multiple forms in which they express knowledge. Specifically related to mathematics education, authors like Ernest (1991), Hersh (1997) and Sierpinska and Lerman (1996) have recognised the epistemology of mathematics as a central concern.

The classical idea is to think of epistemological issues as if they have priority over and thus are detached from semantic ones (Derry, 2013). This hierarchy presupposes a representational view of language as having pre-established, fixed meanings (Brandom, 2008). That is, we already know the meaning of the claims we consider, and the question is merely under what circumstances they can be justified. Adopting an expressivist view means that apart from questions about what would and does justify a claim, one
cannot treat the *meaning* of the claim as having already been settled (Brandom, 2008). Therefore, the semantic and the epistemological are inseparable elements continuously in flux (Derry, 2013). This is precisely what is at heart in the semantic, neo-pragmatic theory of inferentialism (Brandom, 1994, 2000).

According to inferentialism, concepts caught up in statements mean something by virtue of their inferential relations to other statements (Bransen, 2002). They are not mental object-like representations of an external reality (Bakhurst, 2011). Inferentialism “thinks of concepts … as inferential roles” (Brandom, 1994, p. 618) that are conferred with meaning in social practices. To grasp a concept, one must have other concepts, “[f]or the content of each concept is articulated by its inferential relations to other concepts. Concepts then must come in packages” (Brandom, 2000, p. 15). By inferences, we do not primarily mean formal logical inferences, but inferences that are normatively inherent in social languages. For instance, from the statement, “Stockholm is north of Copenhagen” we can infer that Copenhagen is to the south of Stockholm provided the social and cultural norms that regulate the concept’s use allow us to do so (Brandom, 2000).

Since the inferential relations that govern the use of language are normative, they are available for criticism and transformation (Brandom, 2000). It is precisely this transformation of the inferential relations that guides the use of concepts (for example to widen, alter, merge or revitalise conceptual meaning) which characterise translanguaging practices.

Brandom (1994, 2000) like Wittgenstein in his *Philosophical Investigations* (Wittgenstein & Anscombe, 2001) referred to the social practice of language use as language games. Knijnik (2012) used the Wittgensteininan conception of language games to discuss how mathematical language games in rural forms of life in the south of Brazil show resemblance with the globalised mathematics of the school discipline. Knijnik concluded that plural mathematics are constituted by language games whose rules are strongly involved in life culture(s) and they are marked by the rationality(ies) of that(those) form(s) of life, expressed through their own social language(s). As an example, she mentioned a case reported by Knijnik, Wanderer, and Oliveira (2005) in which a peasant explained:

> when estimating the total value of what he would spend to purchase inputs for production, he rounded figures “upwards,” ignoring the cents, since he did not want “to be shamed and be short of money when time comes to pay.” However, if the situation involved the sale of the product, the strategy used was precisely the opposite: the rounding was done “downwards,” because “I did not want to fool myself and think that I would have more [money] than I really had” (Knijnik, 2012, p. 92)

From an inferentialist stance, the rationalities Knijnik mentioned are the implicit inferences inherent in social language (sayings) and in social practices (doings). To make implicit rationalities explicit, we can give and ask reasons for our sayings and doings. That is, we play the language *game of giving and asking for reasons* (GoGAR).

The GoGAR is built around two normative statuses—commitments and entitlements—that emerge in a socially articulated structure of authority and responsibility. When claiming that things are such and such one undertakes a commitment to the claim for which one can be held responsible. A commitment entails not only what is explicitly said but also what follows implicitly from it; that is, by making a claim one also commits oneself to
other normative stands (premises and consequences) that follow from the original claim. For claims to have normative status, they must be normatively appropriate in the social practice in which they are caught up. This involves assessment of claims, hence “there must be in play also a notion of entitlement to one’s commitments: the sort of entitlement that is in question when we ask whether someone has good reasons for her commitments” (Brandom, 2000, p. 43). An entitlement is a commitment that has been acknowledged and endorsed. A way of detecting whether a claim has been acknowledged and hence has the social status of an entitlement is to observe whether the interlocutor uses the claim itself and/or what follows implicitly from it in the interlocutor’s own reasoning. As we participate in social practices, we keep track of all this. We ascribe to each other commitments and entitlements to grasp what we and our interlocutors hold as meaning and knowledge. This track-keeping is about grasping the meaning of what is said (and done) in relation to the normative space in which it is uttered. This is how our semantic consciousness is sensitised.

The GoGAR has two dimensions: a semantic and epistemological what-dimension and a social how-dimension that attends to how we keep track of what other people take to be good and normatively appropriate inferences. Commitments and entitlements make up both dimensions. The what-dimension highlights what implicit inferential relations (premises, consequences and incompatibilities) are embedded in a claim; these guide a person’s commitments and entitlements. The how-dimension highlights how the status of a claim is positioned and changed (a move in the score-keeping) in the GoGAR and is a social matter of responsibility for providing reasons for claims and the social practice of (dis)acknowledging claims.

In a mathematics classroom (and elsewhere), a claim such as “This is a square” occupies positions in a space of reasons. From these positions, knowers of the meaning of the claim can tell what follows from it (e.g. that it is normative and appropriate to say that it has four sides) (Bakhurst, 2011). To be a knower is to navigate ways around normative spaces of reasons, to be aware of what implicitly follows from statements made (Brandom, 1995). Brandom (1995) claimed that

the “space of reasons” … ought to be understood as an abstraction from concrete practices of giving and asking for reasons. The space of reasons is a normative space. It is articulated by proprieties that govern practices of citing one standing as committing or entitling one to another—that is, as a reason for another. What people actually do is adopt, assess, and attribute such standings. (p. 898)

It is from people’s concrete practices of playing the GoGAR that the discursive space of reasons emerges. In a mathematics classroom, this space is shaped by teachers’ and students’ language and mathematics use and by their reciprocal assessment and track-keeping of each other’s use of language and mathematics.

**Epistemological dimensions of language diversity: two illustrative episodes**

The episodes we use to illustrate how inferentialism can uncover some aspects of the epistemological dimension of language diversity come from two different and purposefully disparate contexts. Each episode revolved around a particular concept—the first around *unit of measure* and the second around *half as long*. The first episode took place during
workshops held by Aldo within a settlement of the Nasa indigenous people in Colombia. The second episode has two components. The first is an excerpt of a conversation between Ulrika and Aldrin—a Grade 5 student—who is multilingual. The second component is Ulrika’s investigation that followed her interaction with Aldrin.

**Episode 1**

Episode 1 is based on field notes taken by Aldo as a collaborator in an indigenous project (Caicedo et al., 2009, 2012) conducted by the program of intercultural bilingual education of an indigenous organisation in the Cauca state of Colombia. This project had the aim of researching the mathematical knowledge of the indigenous Nasa culture and supporting the revitalisation of the Nasa indigenous language (Nasayuwe). It is important to contextualise the research as an indigenous initiative, structured by Nasa culture, worldview and ways of validating and registering knowledge. Several meetings were held with community members (elders, healers, teachers, parents and kids) not only to collect data, but mainly to analyse data and validate research results. Following the spirit of solidarity, commonality and collectiveness of the Nasa culture, the process was conducted in several places in an itinerant strategy to cover as many different settlements and territories as possible. Rules for the meeting varied in each settlement according to various types of linguistic proficiency, cultural heritage and political organisation. For instance, in the episode reported here, community members and indigenous researchers spoke mainly in Nasayuwe. Some of the participants were monolingual in Nasayuwe, while others were bilingual in Nasayuwe and Spanish. Aldo is neither indigenous nor a Nasayuwe speaker. During the meeting, indigenous researchers did not find it necessary to make audio-recordings. They only registered the final conclusions and resultant commitments. Therefore, the following history is not a transcription, but a reconstruction from the field notes Aldo took during the meeting.

**Vingette based on Aldo’s field notes from 2008**

In the middle of a workshop organized within an Indigenous Nasa settlement, we were trying to find or create a word in Nasayuwe that can be used as translation of “unit of measure”. I mentioned properties about that concept and provided some examples. In response, a team of Indigenous researchers (all of them were bilingual) explored and discussed possible words in their own language. When they reached a consensus on one term, I asked them if the meaning of that selected word included the property of “unit of measure” of not necessarily being an object of reference, but a magnitude of reference, and they realised that the word did not have such connotations. Therefore, another round of discussions in Nasayuwe took place and they changed the word for another called kxteeçxah. One of the researchers explained to me that both words originally meant other things (it was the expression for “a bunch”), but they estimated that with the last word they could explain better to other Nasa the meaning of “unit of measure”. I realised then, that they had achieved not an equivalent word, but a suitable point of departure to re-create the discussion in the future with other Indigenous people, grasping the concept in their own ways.

This episode shows that to find a word in Nasa that matches the concept of interest—unit of measure—was far from a mere matter of translation. To find a useful Nasa word several things were done. First, the concept unit of measure was placed in a space of
reasons; that is, it’s inferential relations to other concepts were articulated. Aldo did that when he mentioned the properties of the concept and gave examples of units of time and weight. In doing so he conferred meaning to unit of measure by placing it in relation to the other concepts (unit is not an object, but a magnitude) he used when talking. Aldo made the inferential relations between unit of measure and other concepts explicit in his talk; he was engaged in playing the GoGAR. Thereafter, the indigenous researchers placed possible words in Nasa in the space of reasons to articulate their inferential relations to other concepts in Nasa. To find a suitable Nasa word entailed matching enough crucial (for the meaning of unit of measure) inferential relations to other Nasa concepts. When Aldo asked if another property “was included” in the Nasa word (the one that differentiates concrete objects from the quantity of magnitude that the objects have), he made explicit yet more inferential relations between the concept unit of measure and other concepts. The indigenous researchers concluded that a Nasa concept, kxtéeçxah, different than the one they first had in mind, better resonated with the inferential relations of other Nasa concepts that would underpin the meaning of the concept unit of measure in Nasa, according to the Nasa worldview. Thus the indigenous researchers expanded the meaning of kxtéeçxah to include meanings matching unit of measure to be used in contexts where a concept like unit of measure is needed. For the expanded meaning of kxtéeçxah to be used as an entitled claim to knowledge by other Nasa people, new GoGARs will be played, for instance at indigenous schools. The indigenous researchers foresaw the need for new future GoGARs when they explained their refusal to use translations and borrowings in describing the meaning of kxtéeçxah. The indigenous researchers were confident in the capacity of Nasayuwe and the Nasa worldview to express the nuances and particular characteristics of notion of a unit of measure. Episode 1 illuminates epistemological aspects of language use and sheds light on issues of switching between languages not merely as a matter of translation or code switching but also as a matter of discerning rationalities and worldviews inherent in language use.

**Episode 2**

Episode 2 starts with an audio recording of a conversation between Ulrika and a student, Aldrin, when Ulrika acted as a participant observer during a mathematics lesson in a Grade 5 (students aged 11) classroom in Sweden. The Grade 5 classroom was social and language diverse. About a third of the students claimed various degrees of speaking or having access to languages other than Swedish. Aldrin was a second-generation immigrant who spoke Persian and Swedish at home. In an interview made prior to Episode 2, Aldrin had shared that his mother was interested in mathematics and that she provided him with additional mathematics tasks to discuss and work on at home. Thus he had experience of elaborating on mathematical concepts both in Persian and in Swedish. Ulrika’s first language is Swedish. She speaks no Persian. Aldrin was working on a textbook task dealing with the length of the sides of a parallelogram. Aldrin asked Ulrika for help to clarify the task. To Ulrika, he appeared to struggle with grasping the concept half as long.

**Part 1: the exchange between Aldrin and Ulrika**

Aldrin appeared to find no way to relate inferentially the concept half as long to the task of finding the length of the sides of the parallelogram (Table 1). To solve the task, he had to
infer that *half as long* is a matter of relating two lengths (the sides of the parallelograms) to each other in a 1:2 relation. In Turns 2 and 4, when trying to help Aldrin, Ulrika focused on the quantitative part (*half*) of the expression *half as long*, not on the mode or property to which the quantity is connected. In Turn 2, she asked if the sides that are *half as long* are (quantitatively) longer or shorter than the other two sides. In Turn 4, she again asked if the sides that are *half as long* are *longer* than the other sides—if they have a greater quantitative magnitude. She seemed to take for granted that the concept *långa* (*long*) is unproblematic and that the concept use of *long* is fixed, that it mirrors one particular school mathematics *long*. She acted as though Aldrin (or anyone else) could not inferentially relate *långa* (*long*) to other concepts differently than she did. Hence, she ascribed to Aldrin a commitment that excluded the possibility of other inferences than the ones she made herself. This means that in the mathematics-based discursive space that emerged through Ulrika and Aldrin’s conversation, Ulrika’s normative authority use of *long* did not allow the conceptual flexibility needed for her to acknowledge Aldrin’s questioning of the use of *half as long* that he seemed to find normatively inappropriate. She did not entitle him to the claim that troubled the use of *long* because she did not use it in her own reasoning. Fortunately, Aldrin was persistent in his questioning of the use of *half as long*. He started to play the GoGAR. In Turn 7, he said “but why does it say long”; he was now explicitly asking Ulrika for reasons for the textbook’s and Ulrika’s use of *långa* (*long*) by not acknowledging an entitlement to the use of *long* in her previous claims.

Simultaneously Aldrin ascribed to Ulrika a commitment that included the possibility of other inferences than the ones she made, expecting her to provide an answer. Aldrin in his track-keeping detected discrepancies between the inferences he was making from *half as long* and the ones the textbook and Ulrika made. His track-keeping of commitments and entitlements and detection of discrepancies opened up a space in which the use of *half as long* needed be troubled and critiqued in order to generate new knowledge and new languaging (García & Wei, 2014). In Turn 8, this appeared to allow Ulrika to grasp that it was the use of *long*—and not *half*—that Aldrin found troubling. She saw that it was possible to make inferential relations between *half as long* and other claims in ways that convey meanings different from her own. By clarifying this, Aldrin invited Ulrika to share the

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<table>
<thead>
<tr>
<th>Turn</th>
<th>Speaker</th>
<th>Original Swedish</th>
<th>English translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aldrin</td>
<td>Två sidor är … Två sidor … 12.8 och sen de två övriga är hälften så långa [LÄSER HÖGT UR MATEMETIKBOKEN]</td>
<td>Two sides are … two sides … 12.8 and then the two others are half as long [READING ALOUD FROM THE TEXTBOOK]</td>
</tr>
<tr>
<td>2</td>
<td>Ulrika</td>
<td>Mmm … är dom längre eller kortare?</td>
<td>Mmm … are they longer or shorter</td>
</tr>
<tr>
<td>3</td>
<td>Aldrin</td>
<td>Längre!</td>
<td>Longer!</td>
</tr>
<tr>
<td>4</td>
<td>Ulrika</td>
<td>Om de är hälften så långa är de längre då?</td>
<td>If they are half as long are they longer then</td>
</tr>
<tr>
<td>5</td>
<td>Aldrin</td>
<td>Kortare … nå då är de kortare</td>
<td>Shorter … no then they are shorter</td>
</tr>
<tr>
<td>6</td>
<td>Ulrika</td>
<td>Ja …</td>
<td>Yes …</td>
</tr>
<tr>
<td>7</td>
<td>Aldrin</td>
<td>Men varför står det långa … då är det väl … är det inte 12</td>
<td>But why does it say long … then it is not … is it not 12</td>
</tr>
<tr>
<td>8</td>
<td>Ulrika</td>
<td>Vad sa du, varför står det långa är det långa du tänker på då</td>
<td>What did you say, why does it say long is it that long that you are thinking about</td>
</tr>
<tr>
<td>9</td>
<td>Aldrin</td>
<td>Ja typ hälften så långa</td>
<td>Yes kind of half as long</td>
</tr>
</tbody>
</table>

*Note*: In line with Andersson & le Roux (2017)’s, suggestions about ethical research writing about the Other, we account for the students’ talk in Swedish with an English translation provided by Ulrika. Although Ulrika experienced doing the translations as a relatively straightforward matter, she is aware that there are nuances of the languages that may influence her own as well as the readers’ focus on and interpretation of the excerpts.

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Table 1. Excerpt from Aldrin and Ulrika’s conversation.
space with him. In Turn 8, Ulrika seemed to begin to abandon her perception of long as having no other inferential relations than the ones she was making. She asked Aldrin to confirm this which entitled him to articulate that it was the use of long, not half that he questioned. In Turn 9, Aldrin did so. At this point, the mathematics-based discursive space changed. It now encompassed a recognition of the conceptual flexibility needed for a reciprocal acknowledgement of Aldrin’s questioning of the use of half as long and for the assessment and unpacking of the textbook’s claim: “Två sidor är 12.8 centimeter. De två övriga är hälften så långa” (Two sides are 12.8 cm. The two others are half as long).

In this space, as Aldrin’s questioning became entitled it was possible for him to form mathematical beliefs about the sides of the parallelogram, which is precisely what happened as the conversation between the two of them carried on. This shows that the epistemological dimension of Aldrin’s assessment of the textbook claims cannot be separated from the meaning of the concepts at use (Brandom, 2008) in the conversation between Aldrin and Ulrika. Therefore, there is an epistemological dimension in translanguaging spaces. For the mathematics-based discursive space to emerge between Aldrin and Ulrika, Aldrin needed the confidence to question not only the textbook’s use of half as long but also Ulrika’s use of the term. That is, he had to talk back to the stratifying effects of language use (Barwell, 2018) and question Ulrika’s authority as a user of half as long. As a result of him being confident enough to do so, Ulrika became sensitised to the possibility of not merely troubling the use of half but also of long, which was a premise for Aldrin’s formation of mathematical beliefs about the parallelogram. Even though Ulrika is ignorant of Persian, Aldrin’s troubling of the concept moved her into a translanguaging space where they could approach the dynamic and plural language rationalities embedded in their conversation. Using inferentialism, we can unveil the move into this translingual space.

Before we move the analysis further, we present the second part of this episode—Ulrika’s investigation following her interaction with Aldrin—in order to trace some of the inferential relations embedded in the situation.

**Part 2: Ulrika’s investigation**

When she was transcribing the audio recording, Ulrika noticed that Aldrin seemed to find it contradictory that something is half and long at the same time. Ulrika put the Swedish hälften så lång (half as long) into a web-based translation tool and asked for the Persian translation. She then had the Persian translation translated back to Swedish. Now the translation read halv lång (half long) instead of hälften så lång (half as long). From the Swedish concept halvlång (half long), it can be inferred that something is semi-long. Saying that something is semi-long is making a claim that does not put the object that is semi-long in relation to another object; it is not used to make a particular relative relation between two objects explicit. Ulrika asked a colleague who is fluent in Farsi and English for help. She explained that in Farsi the word boland is used for long to talk about a long book, a long time, a long road and so forth. The word boland has its etymological roots in old Farsi language(s). The word for measuring distance or length in Farsi is tool. In Swedish (as well as in English), the words lång (long) and längd (length) are phonologically related, sharing the same etymological roots, but that is not the case in Farsi/Persian, where tool is an Arabic loan word. Taking into account the differences and relations between boland and tool in Persian/Farsi mean mastering what can
and cannot be inferentially derived from each of the concepts. Premises and consequences that follow from using tool have to do with distance and length, which is not the case for boland. When Aldrin questioned the use of long in half as long he initiated a GoGAR that would have to make explicit the inferential relations between the concepts length and half as long. The GoGAR would have to support him in placing the two concepts inferentially related to each other differently from how tool and boland are inferentially related in order for him to grasp the text book use of half as long. Ulrika’s investigation revealed some of the rationalities implicitly embedded in Aldrin’s translanguaging. As Ulrika is ignorant of Persian, she could not play a GoGAR to make this explicit. However, she could play the GoGAR in a way that was sensitive to the language and epistemological rationalities (Knijnik, 2012) inherent in the languages and mathematics at play in the classroom, recognising that she did not yet understand these rationalities.

These two episodes illustrate in their own ways that inferentialism is an approach to multilingualism and plural mathematics that deviates from the language-as-resource metaphor. Our use of inferentialism does not reside in the existence of separate languages or between different mathematics. Rather, an inferentialist approach on translanguaging multilingualism allows elaborations on how people deal with the uncertainties of being understood and of understanding the other as a knower of plural mathematics, by keeping track of commitments and entitlements that endorse inferential relations between concepts. In the episodes, the uncertainties were about how the words kxteçxah and hälften så lång could be used to make mathematical claims to knowledge in Nasa communities and in the Swedish mathematics classroom. In both cases, by using concepts as socially articulated inferential relations, the GoGAR and the normative statuses built around them (used particularly in Episode 2), we have shown that it is not merely a matter of translation or code switching but that there are also epistemological aspects to these uncertainties. Both episodes illustrate translanguaging in which language practices from different cultures and mathematics meet. Inferentialism can capture both the semantic/epistemological and the social dimensions of the discursive mathematics-based spaces of reasons that continuously evolve in such encounters.

**Closing remarks**

In this paper, we began by discussing the first wave of the language-as-resource metaphor in multilingualism in mathematics education to show the extent to which it still belongs to a deficitarian paradigm, where students’ first language is commodified for the needs of the national state (Petrovic, 2005; Ricento, 2005). We then highlighted an emerging second wave of language-as-resource research by highlighting three papers that made efforts to move away from an instrumental view on language use with the purpose of questioning language as representational (Chronaki & Planas, 2018) and theorising both the production of meaning (Planas, 2018) and the sources of meaning (Barwell, 2018) in multilingual mathematics classrooms. We have proposed a shift in research focus from approaches that are concerned with how students use their L1s and informal mathematics to master formal mathematics in L2s towards approaches that embrace translangualism (García & Wei, 2014) and can inform complexities. We suggested that inferentialism is a helpful approach to translanguaging and the continuously evolving new knowledge and languaging it connotes.
We have assumed that the GoGAR is a useful tool—as far as it provides flexible conceptions of concepts—that allows us to analyse the troubling, questioning and expansion of meaning in people’s conversations. At the heart of these conversations are epistemological questions about how a plurality of heterogeneous knowledge meets and how people shape interconnections between them rather than abandoning or capitalising on some of their knowledge (Santos, 2007). The epistemological dimension allows us to study the knowledge that is generated by classroom activities, how articulation of concepts can be developed through the GoGAR, and how students place concepts in the space of reasons.

We used inferentialism in two multilingual translanguaging episodes to illuminate and answer our research question, which was to trace the existence of an epistemological dimension. The report and analysis of data uncovered the dynamic and plural rationalities embedded in face-to-face conversations and illuminated the complexity of emerging mathematics-based discursive spaces. We contend that these issues do not belong within the scope of a language-as-resource approach, in so far as they are not paths to reduce any gap in school achievement. Neither is it a matter of distinguishing between formal and informal language use. Rather, the issues are about shaping a meta-understanding of language diversity that includes epistemological dimensions in which teachers and students do not conceive of mathematics as a fixed and pre-established field of knowledge. Therefore, this emphasis on epistemology differentiates our paper from the current and recent critiques and developments of the language-as-resource approach (Barwell, 2018; Planas, 2018).

With this paper, we contribute to the research and practice of mathematics education in the contexts of language and mathematics diversity as we join efforts to problematise and elaborate on the currently trending approach—the metaphor of language-as-resource. Part of our contribution is our claim that teaching can benefit from being more sensitive to episodes when students provide reasons for concepts or perceive some sayings as contradictory. Such instances provide grounds for reflection about how students and teachers can use concepts and claims as premises and consequences in their reasoning. Within the specificity of multilingual classrooms, we have demonstrated a way for teachers and students to take advantage of episodes of mathematical and communicational uncertainty.

Notes
1. The organisation is called the Regional Indigenous Council of the Cauca (CRIC).
2. This Nasa experience resonates with the attempts of decolonising methodologies proposed by Smith (2013).
3. The same unit of measure can be present in different objects, for instance a stick or a cord that is 1 m long. You can measure with those objects, but they are not necessarily the unit of measure. The unit of measure is the quantity of magnitude present in those objects, e.g. 1 m.
4. For ethical reasons, school location is not mentioned and the student’s name is a pseudonym. Swedish national guidelines for research in the social sciences have been followed (Vetenskapsrådet, 2017).
5. Aldrin named his language Persian while Ulrika’s colleague said she speaks Farsi. To acknowledge their language naming both names are used.

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