POTENTIAL FOR CHANGE OF VIEWS IN THE MATHEMATICS CLASSROOM?

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This paper reports on a study which addressed a question of the potential and perceived influence of the Nordic KappAbel competition on the mathematical views and practices of the participating teachers and students. On the basis of an understanding of “views” and “practices” in the mathematics classroom, the term “didactical contract” is presented and used as a metaphor for structuring the analysis of the data from one of the participating teachers and his students. The problem in the study is closely related to the more general one of the role of external sources of influence on teaching/learning processes in the mathematics classroom.

INTRODUCTION

KappAbel is a Nordic mathematics competition for students in lower secondary school. It is based on collaborative work in whole classes: the class counts as one participant. The competition begins with two web-based qualifying rounds of joint problem solving activity. In Norway, one class from each county continues to the semi-final. Before meeting for the semi-final, these classes do a project on a given theme (in 2004-05 “Mathematics and the human body”). The classes that progress to the semi-finals are represented by four students (two boys and two girls), who are to present their project work at an exhibition and to solve and explain a number of non-routine, investigative tasks. KappAbel, then, focuses on investigations and project work and signals that mathematics does not consist merely of closed lists of concepts and procedures with which to address routine tasks. Also, the emphasis on collaboration in whole classes suggests that there is more to mathematical activity than individuals engaging the development or use of such concepts and procedures. This reflects the aims of KappAbel that are (1) to influence the students’ affective relationships with mathematics (beliefs and attitudes) and (2) to influence the development of school mathematics in line with international reform efforts. The study was conducted in Norway in 2004-05 and sought to contribute to an understanding of the extent to which these aims are met. Hence, the research question we addressed was whether participation in the KappAbel competition has the potential to influence students’ and teachers’ views by influencing the modes of participation in the practices of mathematics classrooms (Wedege and Skott, 2006).

The study includes five types of empirical data, quantitative as well as qualitative: a questionnaire (TS1) administered to the teachers of 2856 grade 9 mathematics classes in Norway, 2004-2005; a questionnaire (TS2) administered to 15 of the teachers whose classes took part in the two introductory rounds of KappAbel and intended to continue with the project work; interviews conducted with eight teachers and six
groups of students; reports and process log books of five classes on the project work of “Mathematics and the body”; and finally observations of 10-15 lessons in 3-4 classes.

Elsewhere we have discussed how we have dealt with some of the conceptual and methodological problems of belief research, for instance the ones of using conceptual frameworks that are not well grounded empirically, of over-emphasising teachers’ views of mathematics for their educational decision making, and that no terminology carries unequivocal meanings (Skott & Wedege, 2005; Wedege & Skott, 2006, p. 48 ff.). In this paper we shall briefly outline our understanding of one of the two key concepts in the study: views. This notion is linked to changes in school mathematical practices by means of a metaphorical use of didactical contract. This conceptual framework presents – together with an inspiration from social practice theory (e.g. Lave, 1988; Wenger, 1998) – a rationale for the KappAbel study.

VIEWS

Belief research has developed into a significant field of study in mathematics education over the last 20 years (e.g. Leder, Pehkonen and Törner (eds.), 2001). One of the recurrent discussions of the field concerns the lack of terminological clarity. There have, then, been many attempts to distinguish between beliefs, conceptions, attitudes, world views and other phrases all of which are meant to capture significant aspects of students’ and teachers’ meta-mathematical orientations, including those related to mathematics teaching and learning.

According to McLeod’s review (1992) of research on affect in mathematics education, “beliefs”, “attitudes”, and “emotions” were used to describe a wide range of affective responses to mathematics. The terms are not easily distinguishable, but the underlying concepts vary along three dimensions. First they differ in stability, beliefs and attitudes being generally stable, while emotions may change rapidly. Second, they vary in intensity, from “cold” beliefs to “cool” attitudes related to liking or disliking mathematics to “hot” emotional reactions to the frustrations of solving non-routine problems. And third, McLeod distinguishes between beliefs, attitudes, and emotions according to the degree to which cognition plays a role, and the time they take to develop. In figure 1, affect in a broad sense in mathematics education is positioned along a spectrum that runs from stability and “cool” on the left (the cognitive end of the spectrum), to fluidity and intensity on the right (the affective end of the spectrum).

Views

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This is not the only analytical description in different dimensions of the affective area in mathematics education research (cf. Evans, 2000:43-45), but in terms of terminological clarification, we find these three aspects and the interrelated characteristics both operational and meaningful. In our terminology teachers’ beliefs include also self-perception (e.g. “Mathematics - that’s what I can’t do” (Wedege, 2002)), aspects of identity (e.g. “In my life, I will never need any mathematics”), and confidence. Attitudes (e.g. “The importance of mathematics is increasing with technological development in society” or “Mathematics is the most terrifying school subject”) are more stable than emotions (e.g. panic or joy). Students’ emotions are seen in connection to personal goals, however some emotions are related to the social coordination, for example when students share similar ideas (Hannula, 2005). Figure 1 illustrates a continuum from cognitive to affective aspects of people’s relationships with mathematics. Using McCleod’s terminology, we study students’ and teachers’ beliefs of and attitudes towards mathematics at the cognitive end of spectrum, and we name these phenomena peoples’ views of mathematics.

“DIDACTICAL CONTRACT”

The interplay – or the social and mathematical interaction – between teacher and students within the frame of the mathematical instruction is crucial in this study, where the topic is the potential change of practices of mathematics classrooms and in the teacher’s and students’ views of mathematics. We found it relevant to involve the metaphor of “didactical contract” in the design of the study and in subsequent analysis because it might combine the emerging school mathematical practices with views of mathematics and of the learning of mathematics. If you want to infer whether views and practices have changed, you need to look beyond the immediately observable actions and organisations of classroom activity. You have to study if and how the mutual expectations of the participants in those practices have changed, i.e. if and how the contract regulating their interactively developed contributions is evolving.

Brousseau’s concept of didactical contract is well known, or at least the term is frequently used. It originates in the framework of the French school of “Didactique des Mathematiques”, and for Brousseau it is inextricably linked to the theory of didactical situations (Brousseau, 1986). Adopting a somewhat different perspective, Balacheff links the didactical contract to the norms for social interaction in a broader sense. To some extent, then, he removes the concept of the didactical contract from the theoretical framework and the empirical studies on which it is originally based, and he defines the didactical contract as follows:
The rules of social interaction in the mathematics classroom include such issues as the legitimacy of the problem, its connection with the current classroom activity, and the responsibilities of both the teacher and pupils with respect to what constitutes a solution or to what is true. We call this set of rules a *didactical contract*. A rule belongs to the set, if it plays a role in the pupils’ understanding of the related problem and thus in the constitution of the knowledge they construct. (Balacheff, 1990:260).

In mathematics education literature outside France, the notion of didactical contract is more often used in this latter, broader sense than in the one closely connected to the theory of didactical situations. This is the use we shall make of the term in the following as well. Thus, our use of the didactical contract does not imply that we import the general theoretical framework of the theory of didactical situations. Rather, we use the term *didactical contract* as a metaphor for the set of implicit and explicit rules of social and mathematical interaction in a particular classroom. The didactical contract, then, in our terminology constitutes the rules of the game in that classroom, rules that on the one hand frame the practices that emerge and on the other are regenerated and transformed by those very same practices.

Among the rules of a didactical contract, three central issues may be addressed: (1) What is mathematics and mathematics education? (2) How do you learn mathematics? (3) Why do you learn mathematics?

The problems in the qualifying rounds of KappAbel are not consistent with a didactical contract in which every student who has read and understood the theory, gone over the examples in the textbook and solved the exercises is expected to be able to solve the problem. The tasks seem to require the students to engage in systematically creative investigation, not supported by the contract just described (see www.kappabel.com for examples).

In the research report, we structured the analysis of the qualitative data by the two first issues addressed by the rules of the didactical contract mentioned above. The results were presented in a “first meeting with the teachers” based on the questionnaire (TS2) and in a “second meeting with seven of the teachers and their students” based on interviews and students’ reports from the competition. In the following section, we present Steinar at the “second meeting” (Wedege and Skott, 2006, pp. 156-161).

**STEINAR AND HIS STUDENTS**

Steinar is a man in his 30s with five years of teaching experience in mathematic. He has a strong background in mathematics and considers himself a mathematician. Through Steinar’s answers to the questionnaire TS2 we get the impression that he emphasises the students’ learning of mathematics that they will need in their everyday life. When he is asked to characterise good mathematics teaching, he allocates almost the same points to discussion and co-operation, to routines and to
project work – yet the least to the last activity. Through the data from two separate interviews, we meet Steinar again together with four of his students, Anders, Stian, May og Toril and the 9th grade project “Mathematics and the Human Body”.

Mathematics is a tool made of rules

The interviewer asks Steinar to describe a normal mathematics lesson. First he explains that they follow the chapters of the textbook and their content rather slavishly: “We go through the constructions on the blackboard, they work with exercises, and I go round supervising them and … that sort of thing.” Then he gets more specific:

Steinar: Yes. … We have three lessons a week. And about fifteen minutes of each lesson I go through what is to be done during the lesson, and what they should finish by the next lesson. … It’s on a Monday so … I give them the entire week’s work schedule that they can work on independently. So in a sense it is optional whether they want to listen to me going through the topics or work on their own. I think most of them listen to me. And then, what I go through, they write down in their rule book. … And then they work on exercises, they sit in pairs and work together. … […] …Next lesson, I also go through a bit of what they ought to get through during that lesson, unless they are already at that point. And I do questions on the blackboard, if someone’s wondering about some question or exercise, if many of them are wondering about the same exercise. But apart from that they do work very independently, they’re a very independent group. They ask each other a lot, and are very easy to deal with (Laughter) (l. 45-60)

During the exposition, Steinar writes one or two rules on the blackboard; he goes through one or two examples and asks the students to write down the rule in their “rule book”. The tasks are from the book, but when the interviewer asks Steinar, whether he has any kind of dialogue with the student while going through the topics on the blackboard, he says:

S: Oh yes, mm. I try to retrieve information they know from before, and … much of it was dealt with last year and the year previous to that, after all. Much of it is repetition so … it’s a matter of getting a dialogue about what they remember from last year up to now, and what they can try to extract from … new things and try to see a connection … yes. Then we often have, or we sometimes, sometimes we have, well it’s not in every lesson, but then we do have a bit of those kind of mathematical brain teasers, where we talk a bit about maths. A bit of that problem solving sort of thing, we have a few of those spicy tasks now and again. … They’re similar to the ones in that Advent Calendar, if you’re familiar with that, with … “matematikk.org” … do you know what I mean?

I: Yes

S: Yes, those types of tasks, and having a bit of talk about them. But forty-five minutes, they pass very quickly so … it’s not always that we have time for all that much of that sort of thing. (l. 74-86)
Steinar uses “brainteasers” and other problem solving tasks to spice up his teaching – something extra, not as part of what he perceives of as the normal teaching of mathematics. “It gets to be a bit too much a matter of routine, I’m starting to think, but I suppose I’ll learn a few new tricks as I go along” (l. 177-178). It sounds as if he considers development and change as coming from outside in the form of tips and tricks. Actually, Steinar calls the KappAbel problems from the qualifying rounds and the class activity around them “a nice interruption of the teaching”.

When the interviewer asked the students, how they experience the similarities between the way they have been working during KappAbel and what they normally do during mathematics lessons, they react as follows:

May: It is maths after all

Toril: Equations and that sort of stuff

Anders: We do have tasks sort of, some places. After all, yes, they’re all tasks, they’re sort of solved in a slightly different way, for example. So in a sense each project is, after all, an exercise. You sort of solve it in a slightly different way. Well, you may have to work a bit more (laughs).

I: Have you ever thought: But, this has nothing to do with mathematics?

Toril: No, I haven’t at least.

Anders: No, not really. At least not about our […], to put it that way. But some of the others were a bit more difficult in terms of discovering the mathematical element. (l. 276-292)

Steinar describes the class as quite homogeneous, and normally the students engage with standard tasks from the textbook listed in their working plan. Steinar encourages a couple of the students to do more challenging tasks explicitly made for the students who do well. Most of them, however, do not take up the challenge.

From the Process Log in the students report, we get the impression that the students do not see mathematical competence and problem solving as the same: it is quite possible to be good in maths without being able to deal with problem solving and vice versa. The class agrees that the four members of the team going to the final “had to be good at maths and able to do problem solving tasks” (Report. p. 5). The character of the KappAbel problems is such that there is not just a single mathematical rule to be followed in order to solve them. According to what seems to be the didactical contract developed between Steinar and his students in this 9th grade, mathematics is a tool made of rules. Problem solving and project work do not appear to be included in the conception of mathematics in this classroom. As Steinar puts it: “… it has been very busy, it’s been a somewhat unfamiliar … way of thinking about maths, bringing it in, integrating it into a topic.” (l. 239-240)
You learn mathematics using the rules to solve tasks

Explaining the teacher’s role in relation to the students’ learning process, Steinar says:

Steinar: Yes, no, well, there are always questions popping up. So I’m going round supervising and listening to how they’re doing and: This is going well. And: You’ve grasped this point and that. And that sort of thing. Of course, I don’t do the problems (Laughter). … Well yes, I suppose I do have a dialogue going with them too … to get a few impressions of what they’ve understood. … Yes, I do have that.

I: But when you’re saying that they work very independently, does that mean that they try on their own first, before they contact you?

Steinar: Yes, they seem to try on their own first and then they mostly ask the person next to them, and then … if they don’t get an answer, then they ask me.

I: Well, exciting, because now …

Steinar: Yes, because I try to be around as little as possible, I want them to try to figure this out a bit by themselves. [inaudible] and then they’ll ask me for an answer and then I’ll say, “have you looked in your rule book?” - No, he hasn’t. “Do that.” Then they’ll look up their rule book, and for the most part they’ll find an answer. So I try to make them even more … independent, too. (l. 121-139)”

And then they look in the rule book ”where they find an answer almost at once”, Steinar says. When the interviewer asks him where he gets the inspiration from for his teaching, Steinar says:

Steinar: … That may well be … what I think is the smartest thing to do, is to go through the topic and then for them to work with exercises. Because it’s … it’s the practise with exercises that makes, that I believe makes, them good at it after all… together with a bit of dialogue. … In terms of performance – and tests – you do, after all, depend a lot on a rule book, so I put great emphasis on good work with the rule book and that they’re able to find the right solution themselves by using it. … […] … I’ve come to understand that the rule book is … terribly important. … which is why I use the approach of going through topics in my lessons, and have them write things down. … Apart from that, I think it’s very important to differentiate the tasks they are given. … (Laughter). (l. 187-198)

From his experience, Steinar knows that it is routine in solving tasks and use of rule books that make the students better in mathematics.

KappAbel

Steinar explains that it was extremely difficult to get the whole class interested in KappAbel. In the end only half of the students (10) participated in this part of the competition, while the rest had ordinary mathematics lessons.
In the students’ Report, it is explicit that project work was a new experience – both to the teacher and the students. However, when the students got down to work they found it fun. They had chosen an open problem on mathematical relations in the human body. In their work, they have only been looking for linear relations (direct proportionality) and they did not find any. However, they also regard this as a result.

Initially, the students did not perceive of the project work as a learning process. In their Process Log they write:

In order for the project to be as efficient as possible, we decided that only half the class would be working on it. If the entire class were to work on the same thing, it would only result in chaos, according to many. In addition, not everyone was equally motivated for working on the project. (Report, p. 5)

This may be understood as if the students consider the product, the exhibition, the competition as the primary purpose of the project. However, from the interview and their report we also get the impression that they have both learnt from the project and enjoyed doing it. In the report they say: ”All in all, we think projects in mathematics are something we ought to do more often. We think our chosen problem is original and slightly amusing.” They elaborate on this in the interview:

I: What has taking part meant to you?

Toril: Meant?

Anders: I think maybe I find maths more fun now (laughs)

Toril: Yes

May: Same here

Stian: Learnt a bit more

Anders: Thinking a bit, being allowed to think a bit differently about mathematics not just being about sitting there looking at your book and writing, sort of /

I: Yes

Anders: They’re slightly different things and sort of ….working together and doing difficult things and trying to manage and cope with it all it’s sort of completely different from sitting there reading from the book.

May: Yes (l. 206-226)

In this 9th grade, teacher and students want the mathematical classroom practices to be different. Steinar wants the teaching to be a little more “spicy” and he often thinks of using computers. It seems like the four students have broadened their ideas of what could be done in the mathematics lessons, but they are not sure if you really learn mathematics doing these kinds of activities (problem solving and project work). Their views of mathematics may not have changed much – problem solving still differs from mathematics, because it requires fantasy, creativity and energy. But they did
have fun and in Anders’ words, the students have been challenged to “think a bit differently about mathematics” when they did the project.

CHANGING VIEWS

We expect neither teachers’ nor students’ views to change easily. Also, following some of the criticism of mainstream belief research, we do not expect teachers’ views to be immediately and uni-directionally related to his or her contributions to the classroom interactions, or to the practices of mathematics classrooms more generally (for a discussion see chapter 2 in Wedege & Skott, 2006). However, views and practices do change and so do didactical contracts in tandem with changing practices and views of mathematics. The question we were addressing in the KappAbel study is if and how this specific mathematics competition may facilitate such change, or if problem solving and project work is to influence only the islands of instruction specifically directed towards KappAbel. In the case of Steinar and his students, the latter seems to be the case, as the dominant views and practices did not change, although there are shifts in attitudes (“we want to do something else”) and emotions caused by the new kind of mathematical activities.

One of the differences between the KappAbel competition and most other initiatives towards reforming mathematics education is the character and sequence of the steps expected to bring about the envisaged changes. Many attempts to bring about change in teachers’ views on mathematics and its teaching and learning go via a pre- or in-service teacher education course. Teachers are expected subsequently to carry their newly established, reformist educational priorities into the schools. In KappAbel the intention is to immediately restructure the teaching-learning practices of mathematics classrooms, by inserting new types of tasks and novel ways collaborating directly into mathematics classrooms.

Classroom practices may change. As we see in the case of Steinar and his students, change, however, is not merely a question of implementing a few new ideas, for instance in the form of tasks that are meant to insert collaborative and investigative elements in mathematics instruction. Change is a matter of teachers and students engaging differently in the activities that mutually constrain and support each other so as to constantly regenerate and further develop the practices of the classroom. As a consequence of their involvement in these changing practices and of their renegotiation of the didactical contract, students and teachers may develop new ways of conceiving of mathematics, of mathematics in schools, and of the teaching and learning of mathematics, i.e. they may develop new views.

REFERENCES


