For a large number of reasons, most of the teachers with whom we work do not have the language of instruction, te reo Māori, as their first language. Teaching in a language, which is an additional language, is not an unusual situation for many teachers around the world. Using data from our ongoing, longitudinal research, we explore the sort of language knowledge that could be included within mathematical pedagogical content knowledge. This has implications for teacher education programs designed for teachers who do not teach in their first languages.

Although there has been research on students who do not have the language of instruction as their first language in mathematics classrooms, there has been much less on the issues faced by teachers who do not work in their first language. In this paper, we draw on data from a longitudinal study in a Māori-immersion school, Te Koutu, to suggest the types of mathematical language knowledge that teachers need for teaching, particularly in situations when they are not teaching in their first language.

Teaching in languages other than their first languages and in which they may not be fluent is a common occurrence for teachers in many countries (Cleghorn, 1992; Bakalevu, 1999). For example, Bakalevu (1999) expressed how as a mathematics teacher who taught in her second language, her lack of facility in English resulted in her teaching in a manner that emphasised procedures.

Looking back on my experience as a mathematics teacher, I remember well the difficulty of helping students make meaning of mathematical problems, particularly at the upper secondary level, where mathematics is more abstract. Some concepts just never registered. My colleagues and I followed the text closely for fear of teaching the wrong thing. Fear of misrepresentation often restrained us from straying away from the given statements and exercises (foreign as they were). When students found it difficult to grasp a point, we faced the problem of finding alternative ways of expressing it while keeping the meaning as close as possible to the original. In the end, against our best intentions, many resorted to drill and practice methods. (p. 64)

For Bakalevu, Fijian, her first language, did not have the political support to be the language of instruction. The decision to only use te reo Māori at Te Koutu came about for different reasons. Since the 1940s, there was a massive shift for Māori people from a relatively stable Māori/English diglossia towards English monolingualism (Benton, 1991). Subsequently, in the 1980s, individuals and communities developed a wave of grassroots initiatives to ensure the survival of Māori language, including the development of schooling in the medium of Māori. As one of these schools, Te Koutu’s policy is that all subjects must be taught in te reo Māori, so the first language of many teachers and students, English, is not used in mathematics classrooms. With this, there is an emphasis on students thinking mathematically by using the mathematics register of te reo Māori to explain and justify their understanding. Teachers as the main agents for ensuring that students
achieve these aims need to have sufficient background both in mathematics and in te reo Māori. Therefore, we see mathematical language knowledge as a component of mathematical pedagogical content knowledge.

**Language as part of mathematical pedagogical content knowledge**

Schulman’s (1986) description of pedagogical content knowledge is often raised when discussing the sorts of knowledge that teachers need. For mathematics teaching, Ball (2000) described pedagogical content knowledge:

Included here is knowledge of what is typically difficult for students, of representations that are most useful for teaching a specific idea or procedure, and of ways to develop a particular idea, for example, the ordering of decimals or interpreting poetry. What are the advantages and disadvantages of particular metaphors or analogies? Where might they distort the subject matter? For example, both “take away” and “borrowing” create problems for students’ understanding of subtraction. These problems cannot be discerned generically because they require a careful mapping of the metaphor against core aspects of the concept being learned and against how learners interpret the metaphor. Knowing that subtraction is a particularly difficult idea for students to master is not something that can be seen from knowing the “big ideas” of the discipline. This kind of knowledge is not something a mathematician would necessarily have, but neither would it be familiar to a high school social studies teacher. It is quite clearly mathematical, yet formulated around the need to make ideas accessible to others. Pedagogical content knowledge highlights the interplay of mathematics and pedagogy in teaching. (p. 245)

Having teachers with strong mathematical pedagogical content is essential if students are to become mathematical thinkers (Kennedy, 1997). Yet, as can be seen in the quotation above, the interaction between the metaphors and analogies used to explain the mathematics can interfere with students’ learning. Shulman (1986) saw the use by teachers of metaphors that aided students’ learning as a vital component of pedagogical content knowledge. In Māori-immersion classrooms, where many students are second language learners, teachers’ knowledge about how to make the most use of the language, including metaphors, impacts on their students’ learning.

Using interview and meeting data from our long running study (see Meaney, Trinick & Fairhall, forthcoming), as well as teacher questionnaires completed in 2010, we suggest how language knowledge is part of pedagogical content knowledge. Many of the teachers were second language learners of te reo Māori and had learnt mathematics and done their teacher education in English. Knowledge of their and their children’s language was raised constantly in discussions about their teaching.

Joubert and Sutherland (2008) in summarising the literature on pedagogical content knowledge stated:

Nowadays there is general agreement in the literature that teachers of mathematics need to know about three aspects of knowledge namely:

- knowledge about mathematics
- knowledge about students’ mathematical conceptions (which includes the development of conceptions)
knowledge about ways of teaching mathematics.

Based on these categories, we provide examples in Table 1 of the language knowledge that we suggest teachers need when teaching mathematics. We do not perceive that this knowledge is only needed by teachers who teach in a language other than their first language. Rather, our contention is that teachers teaching in an additional language are more likely to be aware of the linguistic difficulties that they and their students have. Thus our data set is a rich source for investigation.

Table 1: Language in the pedagogical content knowledge of mathematics teachers

<table>
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<tr>
<th>Pedagogical content knowledge in relationship to language</th>
<th>Examples</th>
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<tr>
<td>Knowledge of mathematical language (mathematics register)</td>
<td>Mathematical terms such as subtraction and grammatical constructions like logical connectives such as ‘and’, ‘because’</td>
</tr>
<tr>
<td>Knowledge about students’ mathematical language (which includes the development of mathematical language)</td>
<td>Do students have sufficient proficiency in the language of instruction as well as the mathematics register to explain and justify what they know and can do?</td>
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<tr>
<td>Knowledge about teaching mathematical language</td>
<td>Second language acquisition strategies to support students gaining fluency in the language of instruction, including the mathematics register</td>
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In the next sections, we provide details about teachers’ awareness of each of these types of language knowledge. The teachers often had suggestions for how they could gain aspects that they felt they were lacking.

Knowledge of the mathematics register

According to Halliday (1978) the register is a set of meanings that is appropriate to a particular function of a language, together with the terms and grammatical structures that express those meanings. Therefore, the mathematics register is the terms and grammatical structures that express mathematical purposes. For many teachers in Māori-medium education, including at Te Koutu, the language of instruction is not their first language. Consequently, knowledge of the Māori mathematics register tended to be learnt on the job. The following extract illustrates one teacher’s experience of this and mirrors the comments made by Bakalevu (1999).

Year 4 Teacher: For my own planning I need to be aware. You plan your unit but with all the reo that’s involved you can’t just go and copy everything. I chased my tail for my āhuahanga (shape) unit. Because I came to you [Uenuku] asking how to do the rotation when I came upon it I didn’t know how to say to them clockwise, anti-
clockwise and all that. So I should have had that at the beginning. If I had that structure instead of running out of the classroom when I saw you coming past. So my classroom practice would mean me being a bit more onto it and going through things and knowing how to say this and this and being ready and since we are all doing the same kaupapa [knowledge] we should all be using them. Perhaps we could do it together a bit more team planning. (Meeting, November 2007)

This teacher could articulate what she saw as an issue: her lack of knowledge of some of the terms in the unit she was teaching on shapes. As well, she was able to suggest strategies for overcoming this. By being better prepared, she would know what vocabulary she needed and so could get help early, either through self-study or from asking someone. She also felt that team planning would assist her. In 2010, other teachers described how they variously used the Māori mathematics dictionary or referred to the curriculum to find appropriate terms. If they encountered a Māori term they did not understand, they used the dictionary again or tried to make sense of it by looking for linguistic clues.

Having a strong background in the language as well as mathematics gave teachers confidence to let students explore ideas with non-standard language. Herbel-Eisenmann (2002) noted that this also occurred in first-language English classrooms where the teacher moved students from using everyday language to official mathematics language. In the following extract from Uenuku’s Year 10 class in 2005, a student invented a mathematical term.

In explaining in her own terms tūtakitanga [point of intersection of two lines], the student used pūwāhi ōrite [the same point]. The teacher, repeated this term to remind students about what they already knew about the points on a line and how this related to the point of intersection. The teacher reused the invented term because he saw it as ‘enabling’ rather than ‘distracting from’ the students’ understanding of the concept.
Making such a judgement requires teachers to have pedagogical content knowledge that includes an understanding of how classroom-generated language supports students’ learning of mathematics.

**Knowledge about students’ mathematical language**

At Te Koutu, many children were second language learners of *te reo Māori* and the their fluency levels impacted on their learning. Part of teachers’ pedagogical content knowledge needed to include the ability to gauge students’ proficiency in using the mathematics register in *te reo Māori*.

Year2Teacher: Yeah, they are quite boisterous, the two young boys, and it's been difficult getting them on task and getting them to learn, really difficult. They are typical boys and that's something I've got to think about in our classroom structure. What do I do, how do I deal with them as a Māori woman and teacher, how do I deal with tamariki [children] like that?

Tamsin: Mmm

Year2Teacher: Because you've got one, the boy that's the most boisterous of all, doesn’t have Māori in his home, doesn’t have the *reo* in his home.

... and consequently that particular child, he sets the whole tone in that classroom because he doesn’t have *reo* at home and while his *reo* has come along really, really well, he is not as articulate as the others.

Tamsin: Yeah

Year2Teacher: So he creates all this...

Tamsin: Because he can't get his point across?

Year2Teacher: Yeah...

Tamsin: So this is a way of getting attention ...

Year2Teacher: Which is what he is doing all the time. Yeah, so I've got to find a way of fixing that.

In the extract, this teacher had gauged the children’s language proficiency and acknowledged that one child’s lack of *te reo Māori* had resulted in him becoming boisterous and disruptive in mathematics and other lessons. However, although she had not determined a way forward, she accepted that a broader approach was needed than just improving his language skills. Although this would be part of what she did, she would also consider her relationship to the child, so that she worked with him rather than trying to change him to fit her expectations.

In 2010, many teachers were able to rate the language proficiency of their students but generally did not group students according to language proficiency in the mathematics classroom. Instead grouping was done according to mathematics ability. While they acknowledged mathematics ability may be connected to the student’s language ability, they were not sure how to manage the different proficiency/abilities of students in the mathematics classroom. This does not mean that teachers ignored the linguistic challenges of their students as discussed in the next section.
Knowledge about teaching mathematical language

One of the main aims that we have had for our research over the years has been to improve students’ mathematical language. Unsurprisingly, in our data many teachers talked about the strategies that they had implemented to support students’ mathematical language development. The following extract is an example from an interview with a Year 6 teacher about the strategies he used to improve students’ written explanations.

What I’ve been producing and role modelling and then getting them to design their own. Then they read all their stuff out and one boy read his out and his answer was quite good which was surprising for him. He said he took it home and his Mum helped him out … in terms of explaining what he was trying to say. Straight away in the question, it was a good example for everyone to follow. Read his out and I made everyone look back at their own to see similarities between theirs and his. The whole of them said no there were no similarities. … Even they could see the value of it to work at their own level.

(Year 6 Teacher Interview/November 2007)

Teachers were not always aware of the linguistic features within *te reo Māori* that could support students learning the mathematics register. *Te reo Māori* has developed the majority of the mathematics register in the last twenty years and many words were chosen to support students’ understanding of the concepts. Christensen’s (2002) research suggested “where teachers understood the Māori origin and mathematical context of the word, acquisition was greatly assisted” (p. 35). Teachers who had not had their own education in *te reo Māori* were not always aware of the reasons why some terms had been chosen. In 2008, two staff meetings, one in June and the other in September, discussed the transparency of the vocabulary. At the beginning of the extract from the June meeting, the discussion focussed on the verb for dividing, *wehe*.

Year 6 Teacher: Are there certain words that help the kids learn what you are trying to teach them? For some words there’s not because you want them to have that wide use of vocab and they are not using that wide use, they are still going back to what they know, so they are not giving you a clear idea of the concept and how deep that knowledge is.

Year 5 Teacher: Mmm, so you are saying that’s not a very good word then?

Year 6 Teacher: Not if you are trying to teach it as division properly to them because *ko wehe* – to make

Year 5 Teacher: *Ko wehe*, ah yeah, *ko wehe kia kitea e hia nga huanga* [divided to see how many elements]

Year 6 Teacher: They’ve got a group and they are trying to make other groups of

Year 5 Teacher: Because another way is to share. *Ae* [yes], and that’s not, *whakawehe* [to divide]

NCEA Teacher:\(^1\): I just think that when you look at the Māori ones there's a lot more, if you just look at the word and you don’t know what they are talking about you could just about work out what it was, you know. The English ones you wouldn’t know what the hell half of them were about. So it’s quite choice when you see

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\(^1\) This teacher taught mathematics to students in the final years of school. He had only begun at Te Koutu six months previously.
things, even like tua ērite [equal number], you are allowed to be same, you know?
That sort of stuff. Tau whānui [range]. All those sorts of things.

These meetings helped the teachers to think about how they were introducing new terms to the children and whether providing an insight into where the terms came from might help the students. When teachers are not first language speakers of the language of instruction, some of the subtleties within that language may not be obvious to them and so not available for them to use in their teaching. Elsewhere, we have documented strategies that the teachers used to support student using the mathematics register (Meaney, Fairhall & Trinick, 2007).

**Implications for teacher education and professional development**

Teaching in an additional language is a common experience for many teachers around the world. Often the decision about what language to teach is made on a political level but it then brings with it practical implications that affect teachers’ work. Yet, there is a paucity of research in this area. In this paper, we have initiated a discussion about the types of mathematical language knowledge that teachers need. Clearly more research needs to be done to further develop understandings in this area.

The mathematics register fulfils a specific function by enabling the expression of a certain set of meanings. All teachers in whatever situation they work in need to understand the role of the mathematics register in mathematics. However, we suggest that this is even more the case when teachers teach in an additional language. In these situations, teachers need more than just the mathematical or the pedagogical knowledge or a combination of these in order to teach.

We have drawn on information provided by teachers in our longitudinal study from a Māori-immersion school to propose three kinds of mathematical language knowledge. Although we have provided only a few examples for each of the new categories, we feel that the data from the teachers teaching in an additional language suggest that it is valid to add these categories to pedagogical content knowledge. Like the original categories, the categories around mathematical language are intertwined. For example, if a teacher does not know the mathematical terms in the language that they are teaching, then they would be unable to gauge the proficiency of their students, nor instigate mathematical language learning strategies.

To cater for the needs of teachers working in an additional language, there needs to be specific teacher education programmes for them. These would enable them to identify aspects of the mathematics register that they need to learn and strategies for doing this. As well, explicit discussions could be held about how to gauge students’ proficiency levels in explaining and justifying their mathematical thinking. It may be that working in an additional language means that teachers struggle to know how to prompt students to explain their thinking, even when they know this is part of good mathematics teaching. Yet it is from listening to children’s talk about their mathematical thinking that their proficiency levels can be gauged. Although as a language learner, teachers may be aware of general language learning strategies this
may not always mean that they know how to transfer this knowledge to mathematics lessons. We have found the mathematics register acquisition model (Meaney, Trinick & Fairhall, 2009) to be a useful starting point for discussions about strategies for improving students’ mathematical language.

Although initial teacher education will be beneficial to many teachers who teach in an additional language, there is also a need for professional development. In our study, teachers who were conversationally proficient in the additional language found themselves teaching in that language, but without knowledge of some aspects of mathematical language. Teachers in this situation will need ongoing support if they are to develop their students’ mathematical understandings.

References


