How does a Government Lower Primary School in India work with mathematics?
- A study on how the teachers’ mathematical beliefs affect the norms operating in the classroom.

Hur arbetar en kommunal lågstadieskola i Indien med matematik?
- En studie på hur lärarnas föreställningar om matematik påverkar normerna i klassrummet
**Foreword**

During this minor study we have had support from a couple of different persons that we would like to say a great thank you to.

First of all we would like to thank the principal, the teachers and the students at Government Lower Primary School in India for taking very good care of us as well as their helpfulness and hospitality for us to be able to go through with this essay. We would also like to say Thank you to Lena Andersson at Malmö University for helping us to get in contact with the school.

We would also like to say thank you to our supervisor Tamsin Meaney for helping us getting this minor study together. Thank you for not giving up on us!

And we would like to send our regards and thanks to our families and friends for supporting us.

While writing this minor study we both have been equally involved in each section.
Abstract
For our study, we visited a Government Lower Primary School in India to inquiry about how a school in another schooling context teaches mathematics. Our research questions were: How does an Indian Government Lower Primary School work with mathematics? What are the teachers’ perceptions of the school’s teaching approach? In addition to these questions and to inquire deeper into this subject, we also investigated How do the teachers’ perceptions and method of teaching connect to Yackel and Cobb’s framework of the different kinds of norms operating in the classroom?

We did a qualitative study, staying at the school for three weeks to interview teachers about their method of teaching mathematics as well as observing how they were teaching mathematics and the norms that operated in the classroom. We also gathered information about their mathematics laboratory.

During our interviews and observations we came to the conclusion that the school worked with activity-based learning by using manipulative materials. All teachers as well as the principal cooperatively strived to meet the curricula objectives, with the same teaching approach. We also found that the teachers’ values and beliefs about how mathematics should be taught, affect the norms operating in the classroom.

This study cannot be generalised for all schools in India or even in this area. This study is a minor study which only considered one particular school which used an interesting teaching method, activity-based learning with manipulatives.

Key words: activity-based learning, manipulative materials, mathematics, mathematics laboratory, sociomathematical norms, teaching mathematics.
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1 Introduction

During our teacher training program we developed an interest in how to teach mathematics in different ways. Through the Swedish International Development Cooperation Agency (SIDA) we got a scholarship for doing a Minor Field Study. The scholarship gave us the opportunity to visit a primary school in India, which seemed to have overcome some of the obstacles of working with mathematics by developing a mathematics laboratory. The students hereby got an opportunity to use manipulative materials to develop their mathematical understanding.

This introduction will present the reason behind this research, followed by information about mathematics in India, the curricula, the school as well as their mathematics laboratory.

1.1 Reason for this research

According to the international PISA (Program for International Student Assessments) (Skolverket, 2013) research on students competence in reading, mathematics and science as well as the Swedish research TIMMS (Skolverket, 2012) – a research on students’ understandings in mathematics and science – mathematics is a critical subject. During teacher training we have experienced that most students have negative attitudes and are experiencing difficulties in learning mathematics. Students state that school mathematics is a boring subject, isolated from the everyday life and they see mathematics as useless (Skolverket, 2012). We believe that a reason for this attitude towards mathematics is due to the teaching methods that are used in today’s school.

In order to reach the many needs of students today, teachers must be prepared with creative and research-based lessons and activities that will match the learning styles of all students. … During the early childhood years, it is very important for children to develop positive attitudes toward math. Teachers can help students develop a positive attitude by providing lessons that engage children and allow them to problem solve by using literature, manipulatives, visualization techniques, and real-world math as well as other strategies during class. (Harwell & Harper 2010, p. 18)

During our years attending teacher education in Sweden, we have discussed a wide variety of mathematical teaching methods. While exploring different teaching approaches, we gained significant knowledge about learning mathematics and came to realisation that profound understandings of mathematical concepts are much more substantial than the memorization of procedures and rules. Suydam and Higgins (1977) concluded from their research that “teaching can be done in rote, mechanical way which emphasises only speed and accuracy. In contrast
teaching can emphasise relationships between different skills and patterns or structures” (Suydam & Higgins, 1977, p. 3).

We consider that an approach focusing on manipulative mathematics entails a thorough understanding of mathematical concepts, especially for number sense and algebra. Much research has been made using such an approach and the majority concludes that it enhances learning of mathematical concepts. In a Swedish literature review, Rystedt and Trygg (2010) suggested that a problem in school mathematics occurs when students use an abstract concept to solve mathematics tasks using memorization alone. Furthermore they emphasise that working concrete with a concept, including materials such as manipulatives, gives students an opportunity to work with different kinds of representations in order to solve mathematical tasks and thereby develop and form thorough concepts. Sfard (1991) emphasises that mathematical conceptions are formed both by means of objects (structurally) as well as within a process; two approaches that she means are complementary. In addition, she highlights the fact that some mathematical concepts are more or less impossible to visualise in a concrete approach. Rystedt and Trygg emphasise the importance of working with manipulatives in mathematics so that students gain a thorough understanding of the mathematics needed within everyday life.

1.2 Mathematics in India
India has a strong mathematical tradition going back hundreds of years. During the last fifty years, India has worked on implementing compulsory school attendance, although the law called Right of Children to free and Compulsory Education Act did not come into force in India until April 1, 2010. Today, education in India is controlled and provided at three different levels which are the central government in Delhi, the state government, and the local sources such as funding and pedagogic help which are largely private (Ramanujam, 2012).

Rampal and Subramanian (2012) wrote that developing curricula for schools as diverse as they are in India is very challenging. It is difficult to ensure the representation of diverse mathematical practices that enable democratic participation and construct a curriculum that benefits all students.

The Right to Education (RTE), an organization that acts for children ages 6-14, recognise the importance that children learn through activities, discoveries and exploration in a child-centred and child-friendly manner. They argue that it takes much effort to implement this teaching approach in schools in India of the reason that most elementary schools do not have a lot of
resources. Many schools do not have access to materials such as games and manipulatives and are hence forced to use textbooks as the only educational material. Another problem is that many teachers do not see the importance or benefits of using these materials (Rampal & Subramanian, 2012).

1.3 Curricula
In this section we discuss The National Curriculum Framework 2005 (NCF), an analysis of the curricula at the central government level. We also used a shortened curriculum which was translated into English by the principal at the school that we visited.

The National Curriculum Framework 2005 divides the mathematics curricula in separate sections for standard one to five. These sections are called the curriculum objectives. The different curriculum objectives are: Geometry (shapes and spatial understanding), Numbers and numbers operations, Money, Measurement (length, weight, volume, time), Data handling and Patterns. Curriculum objectives are developed for each standard. For example, the students in first standard should be able to count up to 100, be able to handle number concepts up to 20 and arrange patterns using different objects. Students in the fourth standard should be able to handle numbers between 1-10 000, do multiplications of four digit numbers by two digits as well as making patterns using geometrical shapes and solids.

In the curriculum there are five guiding principles; connecting knowledge to life outside the school; ensuring that learning shifts away from rote methods; enriching the curriculum so that it goes beyond textbooks; making examinations more flexible and integrating them with classroom life; and nurturing and overriding identity informed by caring concerns within the democratic polity of the county. The guiding principles for mathematics include the importance of identifying, expressing and explaining different patterns, the ability to solve problems, to make connections within the mathematics as well as being able to reason and communicate with support from the mathematic language. The mathematics curricula component stresses the importance of encouraging children to solve problems (National Council of Educational Research and Training, 2007).

1.4 The Government Lower Primary School
The school that our study took place at has the name Government Lower Primary School (GLPS) followed by the name of the village that it was located in. To keep the school anonymous in this
study we therefore decided to name the school Government Lower Primary School (GLPS). This school was involved in a project with SIDA, called *Child Rights, Classroom and School Management*. This project involves schools in India and other countries, where new working methods with a focus on child rights are presented. We made contact with the principal at this school through Lena Andersson, one of our lecturers at Malmö högskola who had been involved in the SIDA project. The reason for going to this school was that they were using a mathematical laboratory and were working with manipulatives.

This school is located in a tribal belt where 55 out of 343 students belong to a tribe or caste. These students do not have the same mother tongue language as the other students and do not show up to school regularly. The other students come from a variety of social and economic backgrounds, overall they could be considered coming from families with an average income. These students had Malayalam as their mother tongue language.

### 1.5 Mathematics laboratory

In the mathematics laboratory housed in a separate room, there were many manipulative materials as well as games. Floors and walls were filled with mathematical illustrations such as shapes, numbers, measuring tools and different mathematical expressions. The room contained a large variety of manipulative materials that were used by teachers as well as students, during lessons. Some of the manipulative materials were used to develop abstract mathematical concepts, whereas other materials were used to develop students’ procedural ability and effectiveness.

Almost all the materials were used in the mathematics laboratory, with some material that was used regularly located in classrooms. Most of the manipulative materials were handmade from recycled materials. They were developed to be adaptable to different learning abilities. Appendix 1 contains pictures and descriptions of the manipulative materials, games and illustrations used in the mathematics laboratory.
2 Theoretical framework

In this chapter, we describe the theoretical framework of our study. The research investigates what happens in the mathematical classrooms and therefore we have chosen to base our study on Yackel and Cobb’s (1996) theory regarding sociomathematical norms. Yackel and Cobb derived this theory during their research on how mathematical values and beliefs among students in a classroom environment were developed so that students “become intellectually autonomous in mathematics” (p. 458). They realised that there was more to it than the cognitive perspective. To make sense of what they experienced, a wider perspective that included the role of interactions needed to be considered. Thus, the concepts of social and sociomathematical norms were developed.

Yackel and Cobb (1996) researched a second-grade mathematical classroom where manipulative materials were used. In this classroom Yackel and Cobb analysed how teachers and students engaged in mathematical activities in the classroom. It seems to be a theoretical framework that we can use to identify the different norms that are situated and created in the classrooms and the teacher’s role in this.

2.1 Sociomathematical norms

According to the National Encyclopaedia (2013) a norm signifies what is “acceptable” or ideal in a social group or convention. Norms are the unspoken, tacit rules that everyone should adapt and conform to.

Yackel and Cobb (1996) derived the sociomathematical norms from the social norms observed in a schooling context. They described sociomathematical norm as “normative aspects of mathematical discussions that are specific to students’ mathematical activity” (1996, p. 458). Sociomathematical norms are unique for mathematics whereas social norms are general and applicable in any subject. In relationship to argumentation, justification and explanation, Yackel and Cobb distinguish between these two norms:

The understanding that when discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm. (p. 461)

These norms are taken-as-shared in the classroom and so “influence the learning opportunities that arise for both the students and the teacher” (McClain & Cobb, 2001, p. 237).
Cobb, Stephen, McClain and Gravemeijer (2011) reflected upon differences between the social and psychological perspectives (see table 1). The social perspective is explained as the way that students act, argue and reason and is connected to the norms operating in the classroom. The psychological perspective on the other hand focuses on the nature of each student’s reasoning and the diversity in the student’s particular way of participating in activities.

Table 1. Social and Psychological Perspectives on classroom learning (Cobb et al. 2011, p. 119).

<table>
<thead>
<tr>
<th>Social Perspective</th>
<th>Psychological Perspective</th>
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<tbody>
<tr>
<td>Classroom and social norms</td>
<td>Beliefs about own role, others´ roles, and the general nature of mathematical activity in school</td>
</tr>
<tr>
<td>Sociomathematical norms</td>
<td>Mathematical beliefs and values</td>
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<tr>
<td>Classroom mathematical practices</td>
<td>Mathematical interpretations and reasoning</td>
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Yackel and Cobb (1996) describe sociomathematical norms as tacit generally accepted understandings of what counts as a different mathematical solution, a sophisticated mathematical solution and an efficient mathematical solution, “similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm” (p. 461). Yackel and Cobb suggest that the different norms are reflexively related:

Both general social norms and sociomathematical norms are inferred by identifying regularities in patterns of social interaction. With regard to sociomathematical norms, what becomes mathematically normative in a classroom is constrained by the current goals, beliefs, suppositions, and assumptions of the classroom participants. At the same time these goals and largely implicit understandings are themselves influenced by what is legitimized as acceptable mathematical activity. It is in this sense that we say sociomathematical norms and goals and beliefs about mathematical activity and learning are reflexively related. (p. 460)

Yackel, Rasmussen and King (2000) also highlight the reflexivity among teacher and students’ mathematical values and beliefs and the classroom social norms. They imply that these norms are constituted, developed and regulated in the classroom. Yackel and Cobb (1996) likewise found that “normative understandings are continually regenerated and modified by the students and the teacher through their ongoing interaction” (p. 474). Students, as well as teachers, continuously participate in these taken-as-shared norms, therefore, they contribute “to the ongoing negotiation of what is taken as normative” (Yackel et al., 2000, p. 277), which results in sociomathematical norms being reformulated. Hence psychological construction of
individual beliefs and values, as well as everyone’s role in the classroom will develop according to the classroom's sociomathematical norms and social norms.

Yackel and Cobb (1996) highlighted the teacher's role as “a representative of the mathematical community” (p. 475). Therefore, the teachers have a responsibility “to make implicit judgments about the extent to which students take something as shared and to facilitate communication” (p. 471). They have an important role “in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students' activity” (p. 475), which contributes to the sociomathematical norms.

These norms are being constructed continuously and are affected by the teacher’s mathematical beliefs, values, knowledge and understandings. Therefore, from the teacher’s beliefs and values about what counts as mathematical “students can develop a sense of the teacher's expectations for their mathematical learning without feeling obliged to imitate solutions that might be beyond their current conceptual possibilities” (Yackel & Cobb, 1996, p. 465). The sociomathematical norms will contribute to different mathematical understandings depending on the implicit mathematical values and beliefs held by the teacher.
3 Aim and questions
The aim of this study is to understand how teachers in a Government Lower Primary School in India teach with activity-based learning using manipulatives. By investigating how learning occurs in another country, as well as another schooling context, we can reflect on and analyse how context affects the teaching of mathematics. This will contribute to discussions about how to work with manipulative and conceptual mathematics, both in Sweden as well as in India. Hence our research questions are:

- How does an Indian Government Lower Primary School work with mathematics?
- What are the teachers' perceptions of the school’s teaching approach?

To be able to inquire deeper into this subject we also want to answer the following question:

- How do the teachers’ perceptions and method of teaching connect to Yackel and Cobb’s framework of the different kinds of norms operating in the classroom?

An additional aim of this study is to raise awareness of internationalization and strengthen international collaboration and networks, and enhance international perspective in education.
4 Related research

Each section from this framework synthesises the research on activity-based learning and the use of manipulatives in mathematics classrooms and is organised using Cobb et al.'s (2011) table (presented in chapter 2 Theoretical framework, page 11) according to the social perspective (social norms, sociomathematical norms and the classroom mathematical practices) and the psychological perspective (beliefs about own role, others' roles and the general nature of mathematical activity in school, mathematical beliefs and values and mathematical interpretations and reasoning). It is important to note that none of the research discussed in this section used Yackel and Cobb’s (1996) theory as a foundation. However, the framework provide a useful structure for considering what work had previous been done in this area. However, we begin by defining active-based learning as well as manipulatives as these two concepts are central to our study.

4.1 Definitions

There are a number of definitions for activity-based learning and manipulative materials. The aim of this section is to synthesise the material and produce our own definitions.

4.1.1 Activity-based learning

From a review of previous research, Suydam and Higgins (1977) described activity-based learning as:

The common element across these conceptions or definitions appears to be student involvement in the process of learning mathematics. This involvement is more than intellectual: the student is actively involved in doing or in seeing something done (p. 1).

In their research, they described activity-based learning as a teaching strategy and indicated that:

activity-based instruction means that the teacher incorporates activities of some type in planning lessons. But under this general umbrella lie a range of specifics:
– The activities vary widely, from actual real-world experiences to working within groups to accomplish a task.
– The activity may serve to motivate; to introduce, to provide reinforcement or practice, to help children apply a mathematical idea to the real world.
– The activity may be integral to the mathematical content or to the instructional objectives, or it may be used pro forma, simply because the teacher believes or is told that the use of activities is necessary.
– The activity may or may not involve the use of objects or manipulative materials (p. 1 f).
In addition to this Smith (1999) emphasised the students’ part in such an approach and outlined the importance of activities actively engaging students to create their own understanding. Smith also mentioned the importance of presenting the mathematical activity so the students become fully engaged. The challenge within the activity could both come from the teacher as well as the student.

Consequently, we define activity-based learning as teaching by means of activities concerning mathematical concepts that engage and involve students in doing, seeing, interacting and communicating mathematics to achieve mathematical understanding.

4.1.2 Manipulative materials
Uttal, Scudder and DeLoache (1997) defined manipulative materials as “concrete objects that are designed specifically to help children learn mathematics” (p. 38). Moyer (2010) elaborated on this to describe manipulative materials as “objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences” (p. 176).

Manipulative materials, therefore, can be considered to be hands-on materials that represent and help students understand different abstract mathematical concepts in a tactile and/or visual way.

4.2 Classroom social norms
This section focuses on research concerning what Cobb et al. (2011) suggest relates to the social perspective, that is, how taken-as-shared tacit statements, developed from teacher's beliefs and values about teaching and learning, form the classroom social norms which are applicable in any subject.

Boaler's (1993) review of previous research was about teaching and learning in classrooms. She concluded that it is important to relate activities to the surrounding context. Making this connection was thought to increase students’ motivation and interest, as well as giving students an opportunity to gain deeper mathematical understanding. Boaler mentioned that connecting mathematics to the everyday life could help students to look beyond the abstractness of mathematics. In comparison, without a context or connection to everyday life, mathematics might be seen as detached and remote from students’ understanding, whereas the use of context could make it more subjective and personal. However, contextualizing teaching and learning is
not unique for mathematics, but applicable to any subject matter. This means that such considerations are related to classroom social norms, rather than socio-mathematical norms.

Smith’s (1999) research took place in middle schools in England. Although his research focused on older students in another context, he highlighted some important obstacles within activity-based learning. Though the majority of research indicates increased learning achievement from using an activity-based approach, Smith's analysis:

suggests that mathematical activities are not enough to achieve learning by themselves; they need to be carried out with a consideration of aspects of presentation, the nature of the pupils' mental activity, the need to ensure pupil reflection and the achievement of socialization of the learning (p. 110).

Thus, Smith's research focused on students’ participation:

Pupils must be actively engaged in constructing their understanding, and ... the activities themselves must be judged mainly by their contribution in assisting pupils to construct their own understanding of concepts selected by the teacher. ... [The activities] must aim to present any mathematical activity in a way that invites pupils to fully engage their higher mental capacities. This can be by the use of a game, a puzzle, a surprise or some other intriguing challenge. In creating a challenge for learners, we must be aware of the need to choose an appropriate level of challenge; one that learners can perceive as offering them a realistic, but not certain, chance of meeting. The challenge can come from the teacher or from the pupil themselves. (1999, p. 109)

Smith went on to state that “many pupils achieve this socialisation of knowledge whilst working from individualised schemes” (p. 110). Furthermore he states that “having developed an understanding of a mathematical concept, there is a need for pupils to be able to communicate effectively about it” (p. 110) which occurs within activity-based learning where interactions are required. Smith emphasises the importance of the teacher, who will facilitate and incorporate the need for communication in all activities and ensures “that pupils are helped to become aware of conventional language and notation” (p. 110).

Though Smith (1999) notes implications for the need of mental involvement, for mathematics in particular, the main substance of his article stresses the importance of socialization, which involves and generates tacit social norms about teaching and learning.

4.3 Beliefs about own role, others' roles and the general nature of mathematical activity in school

In this section, research is discussed which relates to what Cobb et al. (2011) concluded was concerned with the psychological perspective, such as beliefs about roles and the general nature of mathematics. The following research relates to the psychological perspective about students’
beliefs about their own role as well as the role of peers and the teacher in the mathematical classroom.

Using semi-structured interviews about ability grouping in mathematics with 14-16 year old students, Zevenbergen (2003) concluded that “ability grouping create very different learning outcomes for students” (p. 8). This indicates that low achieving students position themselves as dumb or bad whereas the students in the fast-moving groups gain the label of being “the smart ones”. This affects beliefs about everyone’s roles. Using Bourdieau’s ideas, she found that students who are “slow” in mathematics are sooner or later:

being constituted and marginalized by the practices of the field. They are being exposed to practices that exclude them from adding capital to their habitus and are thereby restricted in their future successes and participation in mathematics. There is a sense that they are being shaped by the field as if they have little control of their destinies. (Zevenbergen, 2003, p. 10)

This sets up and emphasises the role of the mathematics teacher as someone who can “add knowledge (or capital) to students”, thereby also

involving a (re)constitution of the habitus of the students. Depending on the background of the students, the primary habitus will facilitate, to greater or lesser extents, their capacity to be constituted as successes or failures within the field. (Zevenbergen, 2003, p. 7)

Though this study looked at older students’ perceptions, it might also have insights for how younger children in other circumstances may experience ability grouping.

4.4 Sociomathematical norms

In this section, research that is related to sociomathematical norms is synthesised. Predominantly, this is research that discusses manipulatives and the way they are used in mathematics lessons, concentrating on the role of the teacher and thus is related to how learning is expected to occur. Puchner, Taylor, O'Donnell and Fick's (2008) as well as Moyer's (2010) research was about how teaching with manipulatives was done, whereas Kelly (2006) focused on how manipulatives were used as a way for teachers to assess children’s learning. To be able to describe the benefits from such an approach the researchers also discussed learning achievement.

Moyer (2001) interviewed and followed teachers that used manipulatives during classes. She concluded that many teachers equated using manipulatives with “fun mathematics”, which gave students a positive interruption from “real mathematics”. The teachers did not see how
manipulatives could help students gain knowledge. Thus, the sociomathematical norms constructed by the teachers did not allow for mathematical understanding to be learnt and taught by means of manipulatives.

As a result of her research, Moyer (2001) emphasised the importance of the teacher’s role. For a teacher to improve and increase student learning with manipulatives, the material needs to represent various mathematical relations. This requires the teachers to facilitate students’ translations between representations, in the form of mathematical objects or actions, into abstract concepts. In this way, students have the possibility to understand the relationship between their prior and new-gained knowledge. Moyer stated “student's own internal representation of ideas must somehow connect with the external representation or manipulatives” (p. 192), implying that the materials will not achieve this by themselves. Thus, the importance of interconnections between concrete and abstract was highlighted.

Similar to Moyer's (2010) conclusions, Puchner et al. (2008) emphasised the importance of the teacher’s role in facilitating “the link between pedagogy and content” (p. 313). They followed four teachers and their use of manipulatives after being involved in a professional development program. They concluded that in three out of four lessons manipulatives were not used as a tool to describe mathematical concepts. In a quarter of the lessons, manipulatives actually prevented student learning. This conclusion indicates that the tacit sociomathematical norm of how mathematical understandings are achieved should be taken as shared. This research shows the benefits of understanding the teachers’ perspectives.

Swan and Marshall (2012) did a quantitative study in which questionnaires about manipulative materials were answered by 820 middle school teachers in Western Australia. Their results suggested that teachers considered that the use of manipulative materials assisted students’ visualization. They also concluded that teachers’ beliefs of how manipulatives should be used affect their implementation. If manipulatives are implemented without clear purpose, the lesson most likely contributes to detrimental learning and misconceptions.

Instead of talking about the importance of teachers’ ways of implementing manipulatives Kelly (2006) describes how to use manipulatives effectively;

Prior to discussing specific strategies, it is important to delineate some of the necessary benchmarks for effective manipulative use. First, it is essential for teachers to realize the impact of referring to manipulatives as tools to help students learn math more efficiently and effectively rather than as toys or play things. If manipulatives are referred to as "toys", students will see them as something to play with rather than as tools to work with to better
understand mathematics. Second, manipulatives must be introduced in a detailed format with a set of behaviour expectations held firmly in place for students to begin to develop a respectful knowledge-base about using manipulatives for math learning. Third, manipulatives need to be modelled often and directly by teachers in order to help students see their relevance and usefulness in problem solving and communicating mathematically. And, finally, manipulatives should be continuously included as a part of an exploratory workstation or work time once open explorations have been completed. (p. 188)

Her research, in which she “explores problem solving in elementary classroom while focusing on how children use (perform tasks) manipulatives … while working on mathematical tasks” (2006, p. 184) has some similarities with the aim of our research. For example, she stressed that “teachers must be able to see beyond obvious correct or incorrect answers into children's thinking processes” (p. 184) thus enable opportunities for students to show their gained knowledge. As well, she highlighted the role that manipulative materials have in promoting students’ communication skills:

> The nature of the manipulative use encourages interaction with not only objects but also with people since it usually involves an action on an object. Being able to learn and use mathematical language effectively helps to lay a strong foundation for conceptualizing and using abstract math skills in everyday life. It also helps students to develop and feel mathematical power as they become more able to articulate, both verbally and in written form, their math thinking processes. (2006, p. 189)

This content emphasises that mathematics requires verbal language, which implies norms unique for mathematics. Therefore, tacit sociomathematical norms indicate that mathematics is taught and understood by means of language and communication.

### 4.5 Mathematical beliefs and values

This section outlines research regarding mathematical beliefs and values, which is related to Cobb et al. (2011) psychological perspective. The research in this section focus on learning achievement connected to the use of manipulative materials.

Ball (2005) stressed that teaching mathematics by means of manipulatives needs clear goals because understanding will not transfer automatically – “although kinaesthetic experience can enhance perception and thinking, understanding does not travel through the finger and up the arm” (p. 47). Furthermore Ball indicated that to gain the benefits from manipulatives, there must be clarification that the manipulatives represent abstract mathematical concepts, which means that discussions and interpretations are a prerequisite. As Ball considered the manipulatives as
connected to beliefs and values about how to teach mathematics, this research can be considered as belonging to the psychological perspective.

Similar to Ball's (2005) suggestions, Uttal et al. (1997) stated:

Although children may learn to perform mathematical operations with manipulatives, they often fail to link this knowledge to more traditional forms of mathematical expression. … the process of establishing a connection between symbol and its referent is not simple. (p. 45)

This, according to Yackel and Cobb’s (1996) conclusions indicates that students as well as teachers’ mathematical beliefs and values will be affected. Uttal et al. (1997) point out that if students are not able to interpret the manipulative materials as a representation of the mathematical abstract concept, they will be required to learn two separate systems; a concrete conception using manipulatives, and a procedural conception where abstract knowledge is memorised. Thus, there is a risk that a concrete approach of working with mathematics will not benefit students’ understandings of abstract mathematical conceptions:

Concrete objects allow children to establish connections between their everyday experiences and their nascent knowledge of mathematical concepts and symbols. In essence, the assumption has been that concrete objects provide a way around the opaqueness of written mathematical symbols. (p. 38)

Uttal et al. (1997) carried out their research on young children’s understanding of the relation between a scale model and the room it represents. They demonstrated by means of an experiment that young children’s understandings of photographs of a room as a symbol of an actual room was similar to their understandings of a physical model representing the room. They found that children had difficulties to interpret the photo as a symbol of the room. Uttal et al. connected these results to the use of manipulative materials, emphasising that manipulative materials do not give a magical advantage for the teacher which in some cases might be the beliefs of the teacher. From a psychological perspective, Uttal et al.’s study indicated that manipulative mathematics could help students to grasp abstract concepts and written symbols, but that

sharp distinction between concrete and abstract forms of mathematical expressions may not be justified. … Manipulatives are also symbols. … successful use of manipulatives depends on treating them as symbols rather than a substitute for symbols. … To learn from manipulatives, children must comprehend how the manipulative represents a concept or a written symbol. (p. 37 ff)

4.6 Classroom mathematical practices

The researchers discussed in this section focused on the mathematical practices taking place in the classroom and emphasise the importance of effective strategies regarding the use of
manipulatives. As these researchers have collected and compared a variety of data, none of which focused on the norms in the classroom, their research is considered as classroom mathematical practices.

Boggan, Harper and Whitmire’s (2010) did a literature review which discusses the positive results of using manipulatives. Additionally the review emphasises that while using manipulatives instructions need to be given in an appropriate and thoughtful way and highlights the importance of discussions. The importance of how the materials are used and the importance of the role of the teacher were also brought up by other researchers, such as Moyer (2001). She conducted research where the importance of how teachers view and teach mathematics was brought up.

Much previous research on manipulatives refers to the work of Sowell (1989) who concluded that there was a small to moderate beneficial effect on student achievement from using manipulatives over a long-term period. For example, Carbonneau, Marley and Selig (2012) used Sowell's research as a base and tried to explore aspects that they believed were missing from her research. Carbonneau et al. found that it was difficult to come to conclusions about this field since as there were many various variables to take into consideration. This statement could indicate that the researchers to some extend are aware that different norms such as the norms in the society, in the school or in the classroom have interfered with the results.

4.7 Mathematical interpretations and reasoning
Suydam and Higgins (1977) are discussed in this section because they focus on what Cobb et al. (2011) equate with mathematical interpretations and reasoning. Suydam and Higgins provided a literature review on learning achievement within activity-based learning in grades K-8. Even though Suydam and Higgins discussed research about particular variables such as learning achievement, we consider that it does not focus enough on how the norms in the classroom affect the results to be considered as social or socio-mathematical norms. In their research, Suydam and Higgins concluded that manipulatives were beneficial for most students because they could be used in an activity-based approach by involving the majority of senses. Suydam and Higgins' definition of activity-based learning is used in the section concerning definitions in the beginning of this chapter. The reason for using Suydam and Higgins’ definition is that there was not a lot of research on active-based learning through the use of manipulatives. When we compared the
different definitions used for activity-based learning we found that their definition defines activity-based learning in a way that is well connected to our research.

Ladberg’s (2000) research did not focus on the use of manipulatives nor on active based learning specifically but is instead situated in the psychological perspective concerning and relating to the brain’s reactions. In her dissertation, Ladberg concluded that brain reactions to different learning situations show that each sense has its own nerve connection. Ladberg argued that by simultaneously activating more senses, several connections between the nerves are built up, which leads to more versatile knowledge that is more resistant to forgetfulness. Ladberg also stated that the brain can assimilate most information when it can create a context from engaging with different activities. This can be linked to Boaler’s (1993) conclusions which were related to the psychological aspects regarding learning achievement and the general nature of mathematics by means of manipulatives and activity-based learning.

4.8 Summary
In regard to our research questions, the sections in this chapter focusing on sociomathematical norms and social norms as well as mathematical beliefs have been the most relevant since they are aligned with and similar to our research. Most of the research mentioned that there are benefits from using manipulatives as well as activity-based learning and emphasised the importance of how the teacher uses the manipulatives in the activities. The two last sections regarding classroom practices as well as mathematical interpretations and reasoning indicated none to a moderate positive effect while using manipulatives and activity-based learning. The research are the ones which have the least direct connection to our research questions because they are based on quantitative studies and do not take the norms operating in the classroom in consideration.

In summary, a lot of the related research regarding manipulatives and activity-based learning that we have found concluded that it was a small to minor benefit using manipulatives. Carbonneau et al. (2012) concluded within their meta-analyses that it is difficult to see if the use of manipulatives is beneficial or not since it depends on many different elements. Additionally Kelly (2006) as well as Moyer (2001) and Puchner et al. (2008) concluded that mathematical understandings could not be transferred automatically, they emphasised the importance of how
the manipulatives were implemented and used during teaching to gain understandings of mathematical concepts.

While looking at the related research we did found a couple of researchers such as Moyer (2001) and Puchner et al. (2008) who did similar research as ours. However, we did not find any research done in a developing country and hence we believe that our study brings up something that has not been discussed before.
5 Method

In this chapter, we explain our process and methodological considerations and justify our choices. We also describe the reliability and ethical principles of our research.

5.1 Methodological considerations
There are two approaches for doing research, the qualitative approach and the quantitative approach. Bryman (2012) wrote that in using a qualitative method the researcher’s aim is to get as many detailed and layered answers as possible whereas the aim with quantitative method is to get answers that can be processed in a faster and clearer way. Stukát (2005) similarly wrote about the opportunity for a more detailed and layered research when using a qualitative method compared to a quantitative one. However, he also mentions the risk that the research becomes subjective and that the results might be affected by the researcher. The qualitative method usually does not use a large amount of data but instead analyses each part of the data in detail, compared to a quantitative research when a wider spectra of data is analysed but not at the same depth.

As our research was based in a specific school and investigated how the teachers at the school worked with mathematics and their perceptions of this method, we adopted a qualitative approach. Backman (2008) stated that a qualitative approach focuses on the individual and thus provides an opportunity to explore how individuals experience and interpret their reality instead of focusing on discovering an objective reality.

Our research is similar with the one made by Puchner et al. (2008) who used a qualitative approach where middle grade teachers’ use of manipulatives in mathematics classes were examined through interviews and observations. Puchner et al. primarily used classroom observations and semi-structured interviews with teachers.

Semi-structured interviews “are used so that the researcher can keep an open mind about the contours of what he or she needs to know about, so that concepts and theories can emerge out of the data” (Bryman, 2012, p. 12). Bryman states that the qualitative interviews are commonly utilised in the qualitative approach because they give flexibility to the interviewer to focus on the interviewees’ own experiences and thoughts. Our research questions were about teachers’ perceptions of their teaching method, so semi-structured interviews were important data collecting method. Bryman (2012) wrote that such interviews should be flexible to match the
different aims of the research as the interviewer has the opportunity to ask follow up questions about what the interviewee says. However, a problem with the semi-structured interviews is that the interviewer forgets what the interviewee answered because the interviewer was thinking of the next question to be asked instead (Johansson & Svedner, 2006). This risk can be minimised by having two persons doing the interviews instead of one. In semi-structured interviews:

Questions may not follow exactly in the way outlined on the schedule. Questions that are not included in the guide may be asked as the interviewer picks up on things said by the interviewees. But, by and large, all the questions will be asked and similar wording will be used from interviewer to interviewee (Bryman, 2012, p. 471).

The interviews were based on our observations and provided us with an opportunity to explore if what we observed was in alignment with what teachers said during the interviews (Johansson & Svedner, 2006). During observations we observed the norms operating in the classroom. In this way, we deepened our understanding for the teachers’ way of teaching mathematics. During observations we used an observation schedule and kept notes as well as video recording and photographing to make it easier for us to expand on our observation notes. Stukát (2005) writes that observations are used when a researcher want to find out more than the interviewee say. From observations, the researcher gets the opportunity to see what teachers actually do. The observations gave us an opportunity to analyse the different norms operating in the classroom using Yackel and Cobb’s (1996) theory of social and sociomathematical norms.

5.2 Process

During our three weeks at the school, we gathered as much data as possible to ensure we were undertaking reliable research. We conducted five teacher interviews and ten observations. The interviews were conducted in pairs in a separate room. Observations were done in the classrooms and in the mathematical laboratory.

We observed and analysed ten separate lessons. Nine of them involved introductions to new topics and seven of these were conducted in regular classrooms. During one lesson, the class worked in groups with the materials in the mathematics laboratory. The main language during the lessons was Malayalam which we were not able to understand. Our observation, therefore, focused on the teachers’ contribution in the lesson, the participation of the students, how they used the manipulative materials and what norms we could see taking place in the classroom. By talking to the teacher about the planning of the lesson, we were able to better understand what we were observing. During the observations, we sat at the far end of the classroom. Doing non-
participatory observations provided a different perspective on how mathematics was taught in the particular classroom compared with if we had been involved in the activities ourselves (Bryman, 1997).

The classroom observations gave us an opportunity to identify how the teaching occurred as well as the students’ engagement. The observations were divided into two different parts. One part was the prepared observations schedule (Appendix 2). At five minutes intervals, we noted what manipulatives were used and when students were involved, as well as who was talking. In addition to the observation schedule, we wrote continuous notes. With the help of our observation notes, we later discussed the patterns we could see. The observation schedules were for example used to calculate on the percentage of time the manipulatives were used. This was done by calculating how many of the five minute segments involved use of manipulatives compared to the total number of intervals.

The school consists of one principal and eight teachers in four different standards, all of whom were interviewed. The interviews were held in English. However, because English is not the teachers’ first language, we provided them with the questions (Appendix 3) the day before the interviews. Although the teachers had the questions the day before, we were still able to ask additional questions. They also requested to be interviewed in pairs, so we interviewed two teachers from the same standard together. However, there was a risk that the teachers affected each other’s responses. During most of the interviews, only one teacher spoke although he or she seemed to answer for both of them. Therefore, in our results one teacher represents both teachers’ opinions. We did a total of five interviews, each of which was between 10-20 minutes long. We decided to give each pair of teachers’ a number from one to four to make it clearer on which pair of teachers that said what. One of the teachers who we mention as teacher 4 spoke a more fluent English and so a lot of the quotes come from teacher 4.

We recorded the interviews with a voice recorder. May (2001) mentions there is a risk that some interviewees might be inhibited when the interview is recorded. On the other hand, recorded interviews makes it is possible for the interviewer to focus on the person being interviewed, which makes it easier to ask follow-up questions. All the interviews were later transcribed. May states that even though it takes a lot of time transcribing, it will be very useful while writing the results and analysis.
Subsequently we looked for common connections within the answers to be able to divide our results into different themes which became the section headings in the chapter 5. After writing the results, we analysed each part using Yackel and Cobb’s (1996) theory on sociomathematical norms, comparing the teachers’ answers in the interviews with what we experienced during the observations.

5.3 Validity and reliability
Since the spoken language at the school is Malayalam, there may have been misunderstandings during our observations that could affect our results and analysis. To reduce this risk, our observations focused on how the teachers interacted with the students and how the students interacted with each other as well as how they used manipulatives. We also spoke to the teacher before and during the lesson to support our understandings of what we observed.

The language barrier could also have affected the interviews as none of us had English as our mother tongue. We hoped that by giving the teachers the main questions in advance we overcame some of these potential difficulties. However, there were some problems during the transcription where we could not understand what the interviewees were saying.

It is also important to be aware of the risk with the teachers being interviewed in pairs. There is a risk that the teachers did not feel that they could express their thoughts of the reason that they were not doing the interview individually.

Additionally our thoughts and beliefs of how mathematics shall be taught might affect and impinge our results as well as discussions.

5.4 Ethical principles
The Swedish Research Council (2010) writes about four ethical principles that should be followed by researchers. The first ethical principle is information requirements which bring up the importance of the participants being informed about the purpose of the study. The second ethical principle is that the participants in the research have the opportunity to withdraw their involvement without any negative consequences. The third ethical principle is the confidentiality requirement which states that participants should be anonymous. The fourth and final ethical principle is the use requirement signifying that the collected data only will be used for this particular research purpose.
These four ethical principles have been taken into consideration doing this research. The teachers, students and parents were informed about the reason for our study and how we were planning to proceed with our research. We also informed the teachers that they during the interview had the possibility to withdraw. When it came to the confidentiality principle we did not provide the name of the school, the teachers name nor the principals. Nevertheless, we did inform them that because we were doing this study through a Minor Field Study-grant, there would be a possibility of others finding out about the school, since it had to be named in the application. The collected data will be used only for the purpose of this study.

5.5 Data analysis

Our data consisted of interviews and observations. We also documented the materials in the mathematics laboratory (Appendix 1). We used the observation schedule to understand what was happening in the classes, which helped us to interpret the other collected data. After transcribing our interviews we identified similarities among the interviews and how they responded to our research questions. For example, several teachers talked about connecting school mathematics to everyday life and this was visible during observations. By going through the observations and the interviews together, we identified the themes discussed in the results. Then, we discussed which of the norms mentioned in Yackel and Cobb’s (1996) theory that were related to each of these specific themes.
6 Results and Analysis
In this section, we describe our results from our interviews and observations. The interviews provided information about the teachers’ views on teaching mathematics. The observations provided information on their work with mathematics in the classrooms. Each result section is followed by a discussion of the norms that were operating in the classroom.

We begin with a description of the teaching approach used at the school, followed by five sections with the themes that arose from the teachers’ descriptions of their teaching and from our observations. The initial section is not analysed against the norms but rather provides a description by the teachers and principal of the teaching method.

6.1 Method of teaching
The principal of the school also teaches mathematics during the students’ Saturday classes and on some other occasions. Thus, her opinions are drawn from her teaching as well as her leading of the school. She explained the school’s approach to teaching, by saying that children worked with concrete materials in the mathematics laboratory so that each group became engaged in the activities. In addition the school used activity-based learning, which she described as students learning by being active and involved in different activities, which included manipulatives. This was very similar to the definition of activity-based learning in chapter 4. She reiterated the importance of the teacher giving the students various materials while exploring mathematics. The principal also mentioned the need for students to be able to influence the direction of the lesson. From her perspective, the teacher should be open to students’ ideas and curiosity and the students should be able to pursue and explore their ideas through working in their own way or in groups. The principal mentioned that the teachers’ role in this method is not to teach or lecture the students on new mathematical skills, but to help the students find and create their own mathematical knowledge.

The principal’s views also drew on the curriculum which was considered important by all the teachers. It reinforced the value of the teaching approach that the school had adopted. Teacher 1 stated:

The new curriculum mentions the aim and methodology of learning as construction of knowledge. Every experience of life is the course of developing a new idea. This means that the child constructs knowledge, they continuously interact with his environment.
Every teacher stated that they were using an activity-based approach. Teacher 4 described that during the introduction to the topic the teachers use the materials from the mathematics laboratory. The teacher planned the lesson so that the students had an opportunity to gain understandings of: number concepts; addition; subtraction; multiplication; and division. Another teacher described the importance of students having the opportunity to create their own knowledge by using concrete materials. During lessons, we observed that the students used materials such as the addition and subtraction boards, bottle caps on a wire as well as measuring instruments (Appendix 1).

Three of the teachers (1, 2, 4) mentioned the importance of using a variety of manipulative materials in activities which were connected to the students’ everyday life. In the observations, examples of these materials were measuring materials and electricity bills. We noted that manipulatives were used by teachers or students around 80% of the time.

To maintain the school’s method of teaching mathematics, the principal explained that the school conducted a lot of workshops where the teachers and the Parent Teachers Association (PTA) cooperated to construct new materials for the mathematical laboratory. By cooperating the teachers had a larger variety of manipulative materials to use during classes. The teachers mentioned that their awareness and curiosity to obtain and acquire knowledge increased as a result of attending these meetings, which they saw as contributing to improving the activities and materials in the mathematics laboratory.

6.2 Student learning
During interviews teachers said that the reason for their teaching method was to give students an opportunity to gain a deep knowledge of mathematics. As was stated in the previous section, they wanted the students to be able to create their own understandings. Teacher 1 said “they construct their own ideas, they made their own ideas in the classroom”. Supporting this, teacher 4 mentioned that “with the participation of the students we cannot simply get the knowledge but they can create the knowledge”, implying the importance of students creating their own knowledge through interactions in mathematical activities, using manipulative materials. She explained “they don’t just learn the point but they get an understanding of the basis of the point”. By this the teacher meant that the students gained an understanding of the mathematical concept.
Teacher 1 also mentioned that a varied, activity-based teaching approach with manipulatives gave students the opportunity to deepen their mathematical understandings.

Teacher 4 emphasised the importance of using the students’ earlier experiences and knowledge to support their learning instead of giving lectures in which the teacher was expected to give students knowledge. This was observed during the mathematics classes, where for example students investigated different information about an electricity bill. In addition to this, the principal stated that this teaching approach “is a good way to acquire mathematical ability and our students then increase the interest and the curiosity in the subject”.

In our observations of the introductions to mathematical topics, it seemed that the students’ main aim was to find solutions or different ways to solve mathematical tasks or problems. Students were mostly given problems which required some investigation, primarily through interactions with others. During one class, we noticed that some students were looking at each other to find or to be able to respond with a correct answer.

Sometimes the teacher required one specific correct answer, but more often the focus was on the explanation about how to get to the answer. The teachers were observed asking questions, facilitating discussions and interacting with students and the manipulatives. The teachers also facilitated the use of mathematical language and the transfer between concrete informal mathematics and abstract formal mathematics (discussed in section 6.4 Concrete to abstract).

6.2.1 Discussion

According to Yackel and Cobb (1996), sociomathematical norms are the take-for-granted ways of acting in the mathematics classroom. Both the teachers’ comments and their observed actions suggest that although they were aware of an alternative lecturing style for teaching mathematics, their beliefs about the importance of active engagement was their taken-for-granted knowledge about how mathematics should be learnt. On the whole, this taken-for-granted knowledge also seemed to be accepted by the students who participated actively in the different tasks. However, the exception was when some students seemed to have given more value to another socio-mathematical norm of needing to have the “correct” answer for the teacher, by writing down another student’s answer. This is discussed later.

By adopting the sociomathematical norm that mathematical knowledge is gained from participating in activities, the teachers manage to do what Smith (1999) suggests is necessary for
creating good learning possibilities, that is working on new, non-routine activities. However, Smith also criticised an approach based on manipulating concrete materials because he considered that students do not automatically learn through activities but needed the opportunity to create a deeper knowledge which would only come from being challenged. Ball (2005) highlighted the significance on how the students are taught in comparison to what they are taught. Thus the sociomathematical norm adopted by the teachers seemed to be in contrast to a common norm in mathematics classrooms elsewhere in the world which implies that learning mathematics is equal to doing as many procedural operations as possible in a short period by means of a text book (Lange & Meaney, 2010).

The need for active participation in mathematics activities also contributes to the acceptance of another sociomathematical norm which values students creating mathematical knowledge. This is connected to a social norm in the classroom about students using their mathematical knowledge to solve problems, so that it is not only the teacher who has the “correct” answers and knowledge. Yackel and Cobb (1996) identified this as a social norm because it is an expectation about what students should do—create and make use of their own knowledge—rather than why they should do it, which would be a sociomathematical norm.

In her article, Ball (2005) emphasised the importance of all students having the opportunity to demonstrate their own understandings of mathematical concepts. She mentioned that for students to be able to apply mathematical knowledge, they needed to have confidence in their own ability. A teacher could contribute to supporting students to build this confidence through making the students aware that their own ideas could contribute to everyone’s involvement and understandings. Yackel and Cobb (1996) implied that if the teacher does not support students being able to utilise their own knowledge, a taken-as-shared norm will be that the teacher is the only authority to judge what correct mathematical knowledge is and students’ main purpose in mathematics lessons is to find the “correct” answer.

The teachers also considered mathematics to be mastered by using manipulatives, which can be considered another related socio-mathematical norm to that of students learn through the use of manipulatives. This norm is in contrast to the one which Moyer (2001) described after doing interviews with teachers, who defined the use of manipulatives as: “fun mathematics”. The teachers explained that they used manipulatives when students were well-behaved, needed a break from the “real” mathematics emphasised in textbooks or at the end of the week when
students deserved something fun. In contrast, the teachers at GLPS saw manipulatives as valuable in developing students’ mathematical understandings and reinforced this with the way that they supported students to engage with them. Moyer (2001) concluded that if manipulatives are seen as toys by the teacher, then this most likely will generate a similarly perception in students. Yackel and Cobb’s (1996) conception of sociomathematical norms implied that teachers’ beliefs and attitudes affect students’ understandings about what is valuable mathematics and how this should be learnt.

Yackel and Cobb (1996) indicated that different sociomathematical norms will contribute to and affect students’ mathematical beliefs and values. Generally, the teachers’ beliefs about students’ ability to find the answers were prominent in our observations. Occasionally, a specific answer was requested by the teacher and this seemed to interfere with the sociomathematical norm of valuing the students’ ability to create their own knowledge.

When the teacher expects a specific answer instead of having the students’ value each other’s explanations and answers there is a risk that the sociomathematical norm among students will become one of finding the correct answer. Yackel and Cobb (1996) emphasised the importance of the teachers’ implicit beliefs and it may be that the teacher’s tacit valuing of a specific correct answers will affect students’ beliefs about what counts as mathematics. Consequently even though we experienced that the teacher endeavoured to highlight the importance of the process to find a solution as well as the correct answer, the students might only come to value the “correct” answer. The students’ acceptance of this socio-mathematical norm may have lead some of them to copy the answer from another student. Researchers, such as Puchner et al. (2008), found that some students copied their neighbour’s answer, instead of finding the connection themselves, suggesting that such a socio-mathematical norm can have a detrimental effect on students’ learning. However, copying seemed to occur rarely and the conflict between socio-mathematical norms did not seem to be too much of a problem at GLPS.

6.3 Contextualizing
The teachers equated mathematical understanding with the ability to solve day-to-day problems. Teacher 4 elaborated on day-to-day problems as “calculating the daily expenditure occurred in the family, purchasing vegetables and other food items from the market”. She described mathematical understanding as:
It means enabling the students to solve problems that could come in their day to day life in the future. And also there are a lot of problems that are related to mathematics in their day to day activities. So and also promote the logical thinking, and reasoning ability and the critical thinking etcetera.

The teachers mentioned that in order for children to be able to solve day-to-day problems, they needed mathematical knowledge, logical thinking, construction of knowledge, problem solving ability, awareness of numbers and the ability for reasoning as well as critical thinking. Teacher 3 mentioned that through working with logical activities the students could create their own mathematical understandings and that students would be able to solve day-to-day problems with the help of their knowledge in mathematics.

Teacher 3 additionally stated that within this teaching method the concepts and objectives that are prerequisite to solve day-to-day problems and various logically activities, strengthen students’ mathematical understandings. Students assimilate knowledge through the learning activities that are connected to their day-to-day life. Similar statements were also done by other teachers at the school. Two teachers (1, 2) expressed that the students learn how to solve problems in their everyday life through using a variety of concrete materials and through participating in activities such as counting money, prices, measuring volumes and length as well as a variety of games. Similar activities appeared in observed lessons where the students worked with electricity bills, calculated and created a fence around a yard and measured the length of the classroom.

6.3.1 Discussion

Teachers’ beliefs about mathematics are concerned with and grounded in everyday life. These values are likely to operate as a sociomathematical norm, in which it becomes taken-as-shared knowledge that mathematics is useful in everyday life. Boaler (1993) also discussed the importance of relating activities to a surrounding context. Making this connection increases students’ motivation and interest, as well as provide students with opportunity to gain a deeper mathematical knowledge. Boaler mentioned that giving students an opportunity to connect mathematics to a context more likely will enable the students to look beyond the abstractness of the mathematics. In comparison, mathematics without a context or basis in everyday life might be seen as detached and remote to students’ experiences, whereas the use of context could be seen as more subjective and personal.
However, there are major distinctions in students’ everyday lives, depending on the context where they live. An everyday life for a student in India might not be similar to the everyday life for students for example in a school in Sweden. It is also important to be aware that the everyday life might not be the same for every student in the same classroom either. Boaler (1993) highlighted that it is important for teachers to keep this in mind. If students are not able to connect to the context that teacher see as belonging to everyday life, they will not be able to perceive the connections. For example when students were working with the electricity bill, the teacher thought that this was a good opportunity for the students to gain the necessary mathematics for a future life. Although the electricity bill seemed to be connected to most students’ future lives, some students may not have electricity in their house and these students would have difficulty seeing how this task was related to a functional real-world context. This might have been particularly an issue for the students with a tribal background. Boaler suggested that teachers needed to be aware of students’ ability to create and develop their own meaning within mathematical tasks, and so there is a need for varied contexts to be used. Depending on the ability to connect the mathematics in a task to one’s own experiences, Yackel and Cobb (1996) indicated that the sociomathematical norm the teacher wants to set up might interfere with the taken-as-shared knowledge among students, since they do not recognise the same situation as belonging to everyday life.

6.4 Concrete to abstract
The teachers mentioned the importance of students being able to create deeper mathematical understanding, highlighting the need for them to have an opportunity to transform knowledge gained from using manipulatives into abstract mathematical concepts. They saw this as supporting students to create deeper knowledge. During observations teachers introduced new mathematical topics with manipulatives by having students handle them. The teachers connected the concrete objects to abstract symbols by reformulating students’ mathematical understandings by recording on the board a symbolic formal mathematical description or by having the students write one in their books.

An example of this was when the students in 4th standard were introduced to perimeter. The students were divided in groups where each group was given a large piece of paper and some rectangular building blocks. Two of the groups were given building blocks of the same size
whereas the third group had smaller building blocks. The students had to measure the perimeter of the paper by means of the blocks. During this process the teachers walked around among the groups and asked questions. Each group found out how many building blocks were needed to measure the perimeter and noted it in their books. Subsequently students were given a new task, to find out the perimeter in centimetres using a ruler. The students once again worked in groups to solve the problem. After a while, the teacher had one student from each group stand in front of the class and show the groups findings. Meanwhile the teacher occasionally facilitated with formal mathematical language. After this was done, one of the students wrote the formula for calculating the perimeter on the board.

6.4.1 Discussion
It would seem that the teachers had set up what Yackel and Cobb (1996) would consider as a taken-as-shared sociomathematical norm in which mathematics was characterised as something learnt by means of manipulative materials. The teachers seemed to connect the concrete experiences to abstract concepts which according to Puchner et al. (2008) are critical. Nevertheless, by linking the symbolic, abstract mathematics to the manipulation of concrete materials, the problems that Puchner et al. (2008) identified in their research did not seem to eventuate. These researchers found that the teachers in their research let the handling of the manipulatives become the purpose of the lessons, to the detriment of developing deeper mathematical knowledge. It seemed that in the GLPS lessons, the focus remained on the mathematical understandings that needed to be developed. Puchner et al. (2008) stated that the teachers needed to plan how to use manipulatives so that the connection between concrete and abstract mathematical ideas became clear to the students. It seemed that GLPS teachers did make students aware of the connection in their own teaching practices. According to Yackel and Cobb, the sociomathematical norm operating in this school and thus these teachers’ classrooms is that abstract mathematical concepts can be mastered using manipulative materials.

However, when teachers are required by a school to adopt a specific teaching approach, there is a risk that the focal point and sociomathematical norms of the school may not be operationalised in the way expected. This is because the teachers’ beliefs about knowledge transfer may interfere with those of the school and lead to some students not being able to make connections between concrete mathematics and symbolic, abstract mathematics. Such problems
were evident in Puchner et al.’s (2008) research when they concluded that some teachers did not manage to support students to see the connections between the concrete and abstract mathematics. As the teachers in Pucher et al’s research were introduced to teaching with manipulatives through a professional development program, they gained the understanding of how to teach from someone else, rather than developing the teaching method from their own ideas. This was similar to what was occurring at GLPS, suggesting that a similar outcome may also be possible. Consequently, teachers’ implicit values and beliefs can affect the taken-as-shared norm among students, regarding how mathematics is learnt through manipulatives (Yackel & Cobb, 1996). However, from the observations it was not clear if this was the case, therefore more research is needed to understand how the interactions between the teacher and the students affect their beliefs.

The teachers gave the students a significant amount of time to work with manipulatives and walked around checking on what the students were doing. The students who finished a task early did not call out nor show their solutions to others. Waiting until everyone was finished or the teacher started the discussion seems to have become a social norm, taken-as-shared knowledge of how to act during all classes, by the students (Yackel & Cobb, 1996). At the end of each part of the lesson, the teacher asked for the solution to the task and then the students called out the answer.

However, as we did not understand the language, we did not know if everybody gained the appropriate understanding from using the manipulatives. It may be that some students ended up learning that the concrete and the symbolic were two separate things, as found by Uttal et al. (2008) in their research. If the students felt that the sociomathematical norms which were operating was that they should give a correct answer to the question, there is a risk even though the teachers thought that the process was the most important part, some of these students may not have made the connection between the concrete and the abstract. Instead they might have repeated what they heard other people answer even if they did not have an understanding of the answer. Thus a clash in perceptions of the socio-mathematical norms between the teachers and the students may not be recognised by either and thus lead to students’ lack of understanding going unnoticed.

6.5 Participation
A reflection made by the teachers was that students showed more engagement and deeper involvement when they used an activity-based method of teaching mathematics that included using manipulatives. The teachers mentioned that the focus of this activity method is to make mathematics an easier subject to grasp, by making it more accessible, convenient and interesting so that students develop an interest in mathematics and mathematical understanding. Teacher 3 highlighted that all students were more active during mathematics lessons as well as in the laboratory, compared to when she was teaching in a traditional lecturing approach. At the same time, this teacher stated the importance of using items that supported the students’ developing mathematical knowledge.

During the observations, students’ engagement, participation and involvement were evident. In the introduction to lessons, at least one student was involved by being at the front of the class performing a dramatization, showing group work or working with the teacher for about 60% of the time. In addition, all students were involved about 80% of the time during introductions. As we did not understand the language, our analysis was based on who was talking to whom for how long and if it seemed to be related to mathematics (as was discussed in chapter 5 on methodology).

Communication had an important role during lessons. Even when there was no introduction to a new mathematical objective or area, students worked in groups to solve tasks and problems, with different concrete materials or games. The tasks were at various levels, with some focused on mathematical understanding of different objectives/concepts, with others involving students doing procedural calculations. The majority of these tasks involved manipulatives and could not be done without the students being active and interacting with each other.

Teacher 4 mentioned that they have such a wide variety of activities at the school giving the teachers an opportunity to take students’ curiosity and eagerness to learn into consideration when devising learning activities. She stated that through this approach “The class becomes more lively”. Three out of the teachers (4, 3, 1) emphasised that this method contributed to students’ full participation. One teacher (3) mentioned that the students seemed more involved when they used manipulatives in the activity-based lesson compared with students who used textbooks or had the teacher lecturing to them in a traditional manner.

The teacher created opportunities to communicate and interact with the students as well as facilitated students’ participation and interactions in discussions through the use of manipulatives.
and various activities. For example, the teacher started a lesson by handing out place value boards as well as place value cards to the students. The teacher then let one of the students say a number giving another student the opportunity to stand in front of the class creating this number on the place value board. After this the next student came and created this number on the place value card and then the teacher wrote this number on the board. After this was done students sat in groups working with their own place value board as well as place value card discussing about the different place-value of the numbers.

6.5.1 Discussion

By giving the students several opportunities to talk during mathematics classes, the teachers established a social norm in the classroom community that learning occurred through communication (Yackel & Cobb, 1996). The teachers stated that through communication students increase their opportunity for participation. This was in contrast to a social norm in which learning was equated with being quiet.

This social norm is supported by research. Smith (1999) concluded that the teacher had a role in making the importance of communication apparent to students. Smith suggested that to facilitate communication, discussion work could be incorporated into activities, which would help students to become aware of the conventional formal language and notations. As was mentioned by teachers at GLPS, Smith stated that when students had an opportunity to communicate about a mathematical activity it would likely contribute to a more fruitful learning environment for each individual. By having this learning environment, the social norm is that students learn mathematics through working in groups and not by working individually. Smith furthermore recognised that the ability to communicate mathematics can neither be achieved by using textbooks nor by working individually and concurrently emphasising the importance of communicating mathematics. At GLPS we did not see any textbooks or much individual work, which supports the idea that the norm in operation was that learning comes from interactions.

The teachers’ valuing of cooperation for exchanging and developing knowledge would be considered by Yackel and Cobb (1996) to be a social norm. At GPLS, this social norm existed across the school, including among the teachers who cooperated to develop their knowledge and methods of teaching in a similar manner to how students worked. Yackel and Cobb stated that teachers play an important role in maintaining, regulating and developing norms. If the teachers
act on a social norm by setting up situations that require students to collaborate, students are likely to accept this as the taken-for-granted social norm. However, we interpreted student communication and participation as a social norm since we did not understand the language and therefore we could not determine what the interactions were about. Thus, the role of communication may be a sociomathematical norm (Yackel & Cobb, 1996) since the discussion was about learning mathematics, rather than learning more generally.

6.6 Multi-level learners
Teachers also acknowledged that the approach was beneficial because they could concentrate on multi-level learners. Teacher 4 stated “In our classrooms there are above average students, average students and below average students. But with this method using these materials we can concentrate on all these three types”. Similarly, teacher 1 mentioned that due to a big variety of activities and concrete materials, all students independently of their level could engage in the activities and thereby get the opportunity to gain understanding.

The principal supported these statements by suggesting that the teacher can provide students with different types of manipulatives in order to implement the curricula objectives. This would generate opportunities for the teacher to differentiate their teaching since the same materials could be used to match each student’s level. She considered that in traditional lecture-style teaching, the teacher may give students the same level of mathematical tasks.

During the observations, we saw how teachers adapted the lessons to multi-level learners. Occasionally students with different competencies or ability levels were divided into the same group and worked together. Teachers also divided the class into groups of students working at the same level. During some lessons, we observed that the teachers had separate introductions for the students working at different levels.

The teachers showed a lot of support for the students by encouraging them when they found a solution to a task and gave them the opportunity to show the solution to the rest of the class. Thus, the teachers highlighted different and sophisticated solutions discovered by the students.

6.6.1 Discussion
By grouping the students by ability level, the teachers developed a social norm about the need to differentiate students according to achievement levels in mathematics. This resulted in a norm
that some students were in need of more support than others, hence accepted by teachers and students.

Even though the teachers, as well as the community, seemed open-minded about discussing the students’ ability level, such an approach has been criticised (Zevenbergen, 2003). Zevenbergen argued that these classifications might induce negative attitudes and a lack of self-confidence towards mathematics. Being a student of an above average level might entail having to operate as a teacher substitute, which also might result in a lack of engagement or interest. She indicated that students who were labelled as low-achieving were likely to be treated as unable to gain mathematical knowledge by the teachers. Therefore, Zevenbergen indicated that a consequence of ability grouping is that “students in these classrooms may be locked into or out of mathematics” (p. 10).

Yackel and Cobb’s (1996) discussion of the importance of teachers’ values and beliefs suggests that if teachers show implicit values which value above-average students’ ability, students who are low achievers may generate a sociomathematical norm, which situates mathematical understanding as something unachievable.

However, our experiences, at this school and in India in general, suggest that there is a great acceptance and open-mindedness about ability levels, without too much focus and attention made to the classification of students. We experienced that students showed a lot of understandings and support for each other regardless of achievement levels, for example we observed students applauding each other for making a great mathematical effort independent of ability level. However as we did not understand the language in the classroom, we cannot be certain how this sociomathematical norm affected students.

In the classrooms there was also a taken-as-shared norm indicating that students, independent of their ability level, benefited from the use of manipulatives. Yackel and Cobb (1996) imply that this becomes a sociomathematical norm which sets up the expectation that mathematical understandings are gained through the use of manipulatives, regardless of ability level. This minimise the risk, mentioned by Kelly (2006), that if teachers emphasise that manipulatives are only need to be used by students who are achieving below average, students may devalue and discontinue the use of manipulatives, thereby reducing their own chances of creating and develop mathematical understanding. Kelly’s conclusions can be linked to Cobb and Yackel’s research
which suggest that teachers’ thoughts affect the social as well as the sociomathematical norms in the classroom.

6.7 Summary
Our analysis has provided us with information about our research questions: How does an Indian Government Lower Primary School work with mathematics? What are the teachers’ perceptions of the school’s teaching approach? and How do the teachers’ perceptions and method of teaching connect to Yackel and Cobb’s framework of the different kinds of norms operating in the classroom?

In response to the first research question, it can be said that the teaching approach in mathematics at GLPS is connected to activity-based learning, were the use of manipulatives is having an important role. The teachers focused on students’ participation and connecting concrete mathematics to the abstract concepts through manipulatives and relevant contexts.

In response to the second question, the teachers’ perceptions of this approach indicated that they believed it would contribute to students being able to create a deeper mathematical understanding. They also believed that this would support the students to use mathematics in their every-day lives. The teachers considered that this method could benefit students, independent of their level and ability of knowledge and indicated on beliefs that students learn by working together. They mentioned that not only teachers, but students possess a lot of knowledge.

The data from the interviews and observations allowed us to respond to the third question by identifying the social as well as the sociomathematical norms operating in the classrooms. Generally when the teacher expressed that a certain method would help students to develop their mathematical understanding, the students seemed to hold these beliefs as well based on how they acted in the classroom. From this we concluded that the norms in the classroom are affected by the teachers’ beliefs and thoughts about how mathematics should be taught. However, when the teachers’ beliefs were different to the sociomathematical norms at the school, we experienced that this affected the sociomathematical norms in the classroom. This means that even though the teacher experienced that there was a certain norm in the classroom, the students might have been experiencing a different norm.
While analysing our result we concluded that what happens in the classroom is affected by the teachers’ thoughts about how mathematics should be taught as well as how they teach mathematics. This has been mentioned by researchers such as Moyer (2001) as well as Puchner, Taylor, O’Donnell and Fick (2008).
7 Conclusion
In summary it can be said that this Indian Government Lower Primary School works in mathematics with an activity-based approach with manipulatives. The teachers believe that this method benefitted students’ participation and ability to develop mathematical understandings. We have also been able to analyse and discuss the various norms that these values and beliefs generated and, thus, the impact they have on the tacit norms among participants in the mathematical classroom praxis.

7.1 Method discussion
After doing our study we came to a couple of conclusions considering our method. One important thing to mention was that we did a qualitative study where we saw how one school in one state works with mathematics. This gave us a great opportunity to inquire more about the methods of this school to get a deep understanding for their way of working. At the same time we found out that the way schools work vary a lot from different schools within the state that we were in and even more if it is compared to other states in India. Therefor it is important to mention that not all schools in India work like GLPS does. We are also aware about the risk that we got too attached to everybody working and being at the school because we were there for three weeks. At the same time we were aware of the importance of being objective.

Before we came to school we knew that there was a risk that the teachers’ level of English might be a problem. But we did not know that they would feel as uncertain as they did. After we started doing our interviews the teachers mentioned that they feel more secure about writing in English rather than speaking English. So, we are aware that we might have got more answers if we had given the teachers a question form to fill in instead. At the same time, we feel that because they got the question a day ahead they had an opportunity to prepare.

Our own thoughts and opinions of how to teach mathematics might have affected our interpretations of teachers’ actions as well as answers during interviews. Consequently, it may have been leading us to focus on those norms of our interest to us during analyses. The fact that the focal point for many of our related research are within a general, quantitative approach, makes it difficult for us to adapt and compare similar research to our study.
7.2 Further research
We feel that there is a need to do more research on how the schools in India work with mathematics through the guidance of the curricula. We believe that the Indian government has come a long way with helping the schools develop different methods of teaching to enable opportunities for the teachers to focus on activity-based learning without using textbooks.

Since this area concerns and is affected by many different variables, there is a lot to investigate. We believe that it would be good to evaluate if the effects, in particular the learning achievement, of activity-based teaching approach with manipulatives is beneficial or not. In addition to this, we suggest that there is a need to conduct a research lasting for a longer period of time, about the implicit mathematical beliefs and values among teacher and its effect on norms operating in the classroom. Similarly, it would be good to know how these norms are regulated and developed within the mathematical classroom with similar circumstances.

The GLPS have been able to hand make manipulative materials from recycled material, making the manipulative very cheap to produce. As the materials are developed to be used in a variety of ways such as various ability levels and ages, we consider that this would be a good and sustainable method for other schools in developing countries. Therefore, we believe that there is a need to do research at more schools like GLPS to gain inspiration. It is important to note that it is not just important how the teachers developed the materials but also how the materials are used.

7.3 Our own development
Through our study in another country and another schooling context we gained the opportunity to widen our knowledge about teaching mathematics. We have been able to experience that it is possible to overcome obstacles such as the cost of materials to adopt an activity-based learning approach, where participation and creating deep mathematical knowledge are a main point. We also had the opportunity to experience how an entire school cooperated to reach the same aim and the positive effects of this, as well as having their method of teaching supported by the government. Through our observations, we experienced how this method activated the students as well as the importance of interactions.

Through our stay at the school we also saw how it is possible to develop materials that cost less than the materials often used in the western world as well developing materials that are
sustainable and can be used in a variety of standards. We do take into consideration the fact that this approach entails a lot of hard work to create the materials, yet we see this as benefitting for our own students.

From the related research and theories, we conclude that the implementation and use of manipulatives as well as implicit mathematical beliefs and values do play an important role in student learning. From the theories about the social as well as the sociomathematical norms we have also become more aware about how our thoughts about mathematics as well as how we teach mathematics affects the students in the classroom.

Additionally doing this study in another schooling context have given us the opportunity to strengthen our beliefs that the use of manipulatives is a good way to teach mathematics and we therefore believe that we through this research learnt a lot that will help us while working as teachers in the future. We are inspired by the school’s way of teaching and believe that we always will have this as an inspiration while working.

This research will engage us in school development questions regarding mathematics in our forthcoming careers.
8 References


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The Mathematics Laboratory

The Government Lower Primary School has during the last two years been building up a mathematics laboratory. The mathematics laboratory is arranged in a separate room where students are able to explore the mathematics in different ways. The materials used in the laboratory are made by the principal and the teachers from recycled and sustainable material. They can be arranged so that they can be used for a variety of ages and knowledge levels. Some of the materials provide students with opportunities to develop their mathematical understanding, whereas other material was developed so that students could practice and automatise procedural calculations in a fun and explorative way.
Place value board and cards

Why?
Place value boards and place value cards are used by the students to gain knowledge and understanding about place value.

How?
- One student decides and describes a number by means of the place value board; each digit in the number is represented by a toothpick or a match. The student put toothpicks/matches in different places depending on each digit’s value.

- Another student/students looks at the place value board to figure out the number so that they can find the right place value cards to produce the same number. Thus to “write” a four digit number the student need to have one place value card to set thousands, one to set hundreds, one to set tens and a last one to set singles.

- After this all involved students write down each part of the number in their writing books, then adding all the numbers together.
  For example: $3000 + 800 + 20 + 5 = 3825$

Variations: Student can, in the same way use a place value board with decimals as well.

Place value boards

Place value cards
Caps on a wire

Why? To gain an understanding of quantity, division, subtraction, addition, multiplication and grouping.

How? Different quantities of metal caps, such as 20, are threaded on a wire. Students put the wire between their feet and use them for different tasks, given by the teacher. The students try to solve tasks with the help of the caps and write down what they do on a paper.

Variation: Different quantity of caps depending on the number. The caps could be divided into groups, divided in twos and threes or used during fractions, multiplication and division.
Pick up sticks

**Why?** For students development in multiplication, addition and subtraction.

**How?** In the game there is one long stick and ten shorter ones; one student starts by dropping the sticks on the ground. The student is supposed to take as many of the small sticks as possible without the other ones moving. The last stick has to cross the longer stick. When the other sticks move the student counts the sticks that s/he has got and determines the points. Then the next students start over.

**Variation:** The sticks can be of different value for example 10 (standard 1), 25 (standard 4), the students can also count their scores by either multiply or add them together.
Appendix 1

**Domino**

**Why?** To practice addition and to develop a good strategy to a good solution to get as many points as possible.

**How?** The students take turns getting the domino cards. They can put the card connected with any other card. Then the student adds the two values of the parts that are combined and that are the students’ points for that round. The students with the highest score after all the cards are used wins the game.

**Variation:** Could use subtraction likewise multiplication.
Snakes on ladder

Why? For students to develop their ability to automate their counting of addition and multiplication.

How? The group of students gets a game board and a dice each. The students take turns in throwing the dice. You start the game when you get a one. Then, every time you throw the dice you walk the number of steps shown on the dice. Every time, the student writes what s/he counted before he can move that number of steps. If you end up in a square/path with a ladder you follow the ladder to another square, whereas if you end up on a snake you fall down. For example: one student throws the dice, showing 8. The student’s counter is standing at number 26, the student has to do the addition before it is moved.

There is one snake game with different multiplications. When you land on a step with a multiplication, you move that many steps after you have calculated the answer.

Variation: Variety can be achieved with dices with different values/digits. The students use different kinds of board games with numbers 1-99, 901-1000……. They could also multiply the numbers of the dice with the number that they are on.
Stitch the pattern

Why? Provides students with opportunities to develop their ability to automatise procedural calculations.

How? The students get three cards involving addition, subtraction, multiplication and/or division. The students calculate the tasks on each card to find the right number on the sewing card and sew the answers. When students are done, a pattern/picture will be produced, so that the teacher and the student easily can see if the calculations were correct.

Variation: The same sewing card could be used in combination with different counting cards for a variety of difficulty.
Jigsaw Puzzle

Why? To practice multiplication, division, addition and subtraction.
How? The students get a frame and puzzle-pieces. On the back of the pieces the task is written and the answers are written on the base of the frame. The students need to put each puzzle piece on the correct spot, which requires calculating the task on each piece.
Variation: Different numbers can be used, different quantity of pieces. The same puzzle could be used at different levels depending on which puzzle-pieces being used.
The spinner

**Why?** To practice calculations and managing to find the highest/lowest number.

**How?** The students take turn in spinning two spinners. Hence they add, subtract, multiply or divide these two numbers depending on the teacher’s instructions. The person that gains the highest answer gets 10 points. The spinner has six sides and is threaded on a stick with number on the back and front of the hexagon so that it can be used at a variety of levels.

**Variation:** This game could be played on different levels depending on the number range on the spinner and which calculations were used.
**Number Puzzles**

**Why?** To develop problem solving methods as well as number sense.

The number of the corners on each square should add up to 34, placing the numbers 1-16 in the corners. Each number can only be used once.

Each row as well as each circle should add up to 65, using the numbers 1-25.

The students can use number 2 three times, number 3 three times and number 4 three times. Every row as well as the diagonal should add up to 9.

Placing the numbers 1-9 in each square to find a sum that will be equal both across, vertical as well as diagonal.

For older students you could use higher numbers.
Appendix 1

The answer on each row, diagonal and level should add up to 42 and the students can use the numbers 1-27

The “star” has eight corners; seven beans should be placed in a corner each. To place it there the students need to use the printed diagonal/lines. You are not allowed to start where there is a bean, because that entrance is locked.

HINT! Always put the bean where you the time before started moving the bean from.
Surface area blocks

**Why?** For the students to develop a thorough understanding for mathematical concepts such as geometry, surface area and fractions.

**How?** The students use the blocks in different tasks such as measuring the surface or perimeter of a rectangle or different shape. Calculate the quantity that will be needed to build the shape with walls. In the laboratory, shapes were painted on the floor.
Materials for introductions

In this appendix are descriptions of some of the materials in the mathematics laboratory that were used during the mathematics lessons in the classrooms.

Place Value board

These materials were used during the introduction to place value. The one to the left was used by students to create a number. This is used so the students will get more certain of where to place the ones, tens, hundreds and thousands. After the first student has created a number on the place value board the second student creates the number by placing the circles on the sticks.

Addition and subtraction board

This board is used to describe and make addition and subtractions concrete. For example with an addition, the numbers are placed in the compartments and later added together. The solution will be shown in the bottom row. In this way the students get the opportunity to understand what happens when the students are calculating.

Angle at the door
Weight and Volume

Geoboard
<table>
<thead>
<tr>
<th>Time</th>
<th>Activity Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 min</td>
<td>Läraren pratar med alla elever/Teacher talk to all students</td>
</tr>
<tr>
<td>10 min</td>
<td>Läraren pratar med en grupp elever/Teacher talk to a group of students</td>
</tr>
<tr>
<td>15 min</td>
<td>Läraren pratar med enskild elev/Teacher talk to individuals</td>
</tr>
<tr>
<td>20 min</td>
<td>Lärare använder lab. Mat eller dramatisering/Teacher use manipulatives or dramatizing</td>
</tr>
<tr>
<td>25 min</td>
<td>Elever samtalar i grupp/Students talk in groups</td>
</tr>
<tr>
<td>30 min</td>
<td>Elever samtalar i par/Students talk in pairs</td>
</tr>
<tr>
<td>35 min</td>
<td>Elever är delaktiga i genomgång (visar) / students are a part of the introduction by being in front of the class</td>
</tr>
<tr>
<td>40 min</td>
<td>Elever svarar på frågor från lärare/Students answer questions from the teacher</td>
</tr>
<tr>
<td>45 min</td>
<td>Elever arbetar enskilt/Students work separate</td>
</tr>
<tr>
<td></td>
<td>Elever arbetar i matematik böcker/Students talk in mathematic books</td>
</tr>
<tr>
<td></td>
<td>Elever arbetar med samma laborativa metrial/Students work with manipulative materials</td>
</tr>
<tr>
<td></td>
<td>Elever arbetar med individuellt laborativa material/Students work individually with manipulative materials</td>
</tr>
<tr>
<td></td>
<td>Elever arbetar med lab. mat. i grupp/Students work with manipulative materials in groups</td>
</tr>
<tr>
<td></td>
<td>Elever arbetar med lab. mat. Enskilt/Students work with manipulative materials alone</td>
</tr>
<tr>
<td></td>
<td>Elever löser uppgift enskilt/Students solve the tasks individually</td>
</tr>
<tr>
<td></td>
<td>Vilken typ av lab. mat/ What kind of manipulative materials are being used</td>
</tr>
<tr>
<td></td>
<td>Övrigt/Other</td>
</tr>
</tbody>
</table>
**Principal**

What made you commence this approach?

Where were any difficulties within the startup of the method/project?

Pros within this approach? How? In what way?

Cons within this approach? How? In what way?

What does the curriculum say according to teaching methods?

How was this method/approach implemented amongst the teachers?

How is the school evaluating this teaching approach? /What is being done to monitor the situation?

How do you perceive the teachers apprehensions of this teaching method?

How do you perceive the students apprehensions of this teaching method?

How did you proceed with this mathematics workshop – project?

Other questions

**Teachers**

Can you tell us about how you teach mathematics?

How do you feel of this way of teaching?

What are the positive things with this method?

Have you find any problems or implication, obstacles within this teaching method?

How does using the concrete materials, that method, relate to the curricula?

How was this implemented amongst you teachers at this school?

How do you think that the students think about this method?

Can you see a difference in student’s involvement and understanding with this approach?

What does a mathematical understanding means to you?